Mathematical literacy

Definition and building blocks

DEFINITION OF MATHEMATICAL LITERACY

Mathematical literacy is a person's ability to employ mathematical thinking and mathematical knowledge in order to:

- Use mathematical concepts, procedures and tools in differently structured environments;
- Analyse, substantiate and effectively communicate his/her ideas and results when formulating, solving and interpreting mathematical problems in differently structured environments;
- Perceive and be aware of the role of mathematics in everyday and professional life; connect it with other areas and make responsible decisions based on mathematical knowledge; be willing to accept and co-create new mathematical findings

List of abbreviations

MP – mathematical literacy

NA-MA POTI – Scientific and Mathematical Literacy:
The Development of Critical Thinking and Problem-solving

BUILDING BLOCKS OF MATHEMATICAL LITERACY

1st BUILDING BLOCK

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.1. Understands messages with mathematical contents.

1.2. Knows and uses technical terminology and symbolism.

1.3. Presents, substantiates and evaluates his/her own thought processes.

1.4. Recognizes, understands and uses mathematical concepts under different circumstances.

1.5. Knows and uses appropriate procedures and tools under different circumstances.

1.6. Predicts and assesses results, substantiates claims, procedures and decisions.

1.7. Uses different strategies when solving mathematical problems.

2nd BUILDING BLOCK

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.1. Discusses diverse real life problems
(problems that do not require mathematical modelling).

2.2. Discusses situations using mathematical modelling:

    2.2.1. Transfers a situation to the mathematical context;

    2.2.2. Designs mathematical models for the given situation;

    2.2.3. Uses mathematical models;

    2.2.4. Evaluates mathematical models.

2.3. Understands mathematical practices in different contexts.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.1. Understands messages[1] with mathematical contents

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) (Receives) Understands simple oral, graphic messages with mathematical contents; b) Sums up a message with mathematical contents and answers questions; c) Independently obtains information from oral sources.	a) (Receives) Understands simple and structured messages with mathematical contents; b) Uses simple reading strategies during comprehensive reading of mathematical texts and when solving word problems; c) Sums up a message with mathematical contents, extracts the essence and the required information; d) Independently obtains information from oral and written sources.	a) (Receives) Understands simple and structured messages with mathematical contents; b) Uses simple and complex reading strategies during comprehensive reading of mathematical texts and when solving word problems; c) Sums up messages with mathematical contents, extracts the essence and the required information, and creates a new message; d) Independently obtains information from oral and written sources.	a) (Receives) Understands simple, structured and complex messages with mathematical contents; b) Uses appropriate reading strategies during comprehensive reading of mathematical texts and when solving word problems; c) Sums up messages with mathematical contents, extracts the essence and the required information, and creates a new message; d) Independently obtains information from credible sources.	a) (Receives) Understands simple, structured and complex messages with mathematical contents; b) Uses appropriate reading strategies during comprehensive reading of mathematical texts and when solving word problems; c) Sums up messages with mathematical contents, extracts the essence and the required information, and creates a new message; d) Independently obtains information from credible sources.

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1 **Message:** people communicate with one another by conveying messages using various symbols (e.g. spoken language, gestures, body language, images, audio and light signals, written texts, etc.). In the communication process all the participants receive, send/create and interpret messages which are connected for a specific purpose; communication is always a two-way process as it involves the simultaneous detection and exchange of messages on both sides.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.2. Knows and uses professional terminology and symbolism

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Recognizes professional terminology in a message and understands its meaning; b) During activities and concrete presentations of mathematical concepts, the learner names and describes concrete or graphic representations (shapes, solids, numbers, quantities, relations, colours, position).	a) Recognizes professional terminology and symbolism in a message and understands their meaning; b) Names and describes mathematical concepts using mathematical terminology and symbolism; c) Uses mathematical language[2] to describe a mathematical situation.	a) Recognizes professional terminology and symbolism in a message and understands their meaning; b) Writes a verbalized (simple) mathematical message using mathematical symbols and vice versa (reads/verbalizes a text containing mathematical symbolism); c) Uses appropriate terminology and symbolism to describe mathematical objects and structures, and their properties; d) Uses mathematical language to describe a situation; e) Understands the different meanings of individual mathematical terms and symbols.	a) Recognizes professional terminology and symbolism in a message and understands their meaning; b) Writes a verbalized mathematical message using mathematical symbols and vice versa (reads/verbalizes a text containing mathematical symbolism); c) Uses appropriate terminology and symbolism to describe mathematical objects and structures, and their properties; d) Formulates definitions in simple mathematical situations and uses them; e) Sensibly uses mathematical language in other contexts as well; f) Understands the different meanings of individual mathematical terms and symbols, and uses them flexibly.	a) Recognizes professional terminology and symbolism in a message and understands their meaning; b) Writes a verbalized mathematical message using mathematical symbols and vice versa (reads/verbalizes a text containing mathematical symbolism); c) Uses appropriate terminology and symbolism to describe mathematical objects and structures, and their properties; d) Formulates definitions in mathematical situations, knows their purpose, and uses them; e) Sensibly uses mathematical language in other contexts as well; f) Understands the different meanings of individual mathematical terms and symbols, and uses them flexibly.

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2 **Mathematical language:** we use it to name or put into words mathematical concepts, objects, structures, etc. using technical (mathematical) terminology and symbols.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.3. Presents, substantiates and evaluates his/her own thought processes[3]

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Orally presents the process of solving tasks, and talks about his/her findings and thought process; b) Gets involved in a conversation about mathematical situations; c) *Assesses his/her own work according to the set guidelines.*	a) Appropriately presents the process of solving tasks and problems, and talks about his/her findings and thought process; b) Participates in a mathematical discussion; c) *Assesses his/her own work according to the set criteria[4].*	a) Appropriately presents and explains the process of solving tasks and problems, and mathematical thinking; b) Participates in a mathematical discussion; c) *Assesses his/her own work according to the set criteria.*	a) Appropriately presents, explains and sums up the process of solving tasks and problems, and mathematical thinking; b) Participates in a mathematical discussion; c) *Assesses his/her own work according to the set criteria.*	a) Appropriately presents, explains, substantiates and sums up the process of solving tasks and problems, and mathematical thinking; b) Participates in a mathematical discussion; c) *Assesses his/her own work according to the set criteria.*

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3 **Thought process:** it is triggered by situations and we are aware of it only to an extent; through practice or reflection we try to become aware of our own thoughts when solving tasks; our emotions also trigger thoughts and are reflected through our behaviour, affecting our perseverance, our view of a task as a challenge, etc.

4 **Criterion:** is a "measure of success" that helps us to assess and be aware of our own knowledge and the fulfilment of learning intentions; we use it to define the important aspects of knowledge, understanding, abilities, skills.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.4. Recognizes, understands and uses mathematical concepts[5] under different circumstances

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Recognizes a concrete object, an image of an object for presenting a mathematical concept; b) Recognizes the mathematical concepts represented in different ways (verbally, concretely, graphically) in known situations; c) Illustrates a mathematical concept with a chosen representation; d) Finds common properties and differences between concrete and graphical representations of a mathematical concept.	a) Recognizes the mathematical concepts represented[6] in different ways (concretely, graphically, symbolically) in known situations; b) Uses different representations of mathematical concepts and switches between them; c) Finds common properties and differences between individual representations of a chosen mathematical concept; d) Can visualize sizes[7] and quantities.	a) Recognizes the mathematical concepts represented in different ways (concretely, graphically, symbolically) also in less known situations; b) Uses sensible representations of mathematical concepts and switches between them; c) Uses examples to confirm or reject claims about the properties of mathematical concepts; d) Can visualize sizes and quantities; e) Differentiates between mathematical concepts based on their properties and the relations between them; f) Interprets different (similar) situations using mathematical concepts.	a) Recognizes the mathematical concepts represented in different ways (concretely, graphically, symbolically) in different situations; b) Uses sensible representations of mathematical concepts and switches between them; c) Uses examples to confirm or reject claims about the properties of mathematical concepts; d) Can visualize sizes and quantities; e) Differentiates between mathematical concepts based on their properties, and recognizes similar concepts and the relations between them; f) Interprets different situations (including new ones) using mathematical concepts.	a) Recognizes the mathematical concepts represented in different ways (concretely, graphically, symbolically) in different situations; b) Uses sensible representations of mathematical concepts and switches between them; c) Uses examples or counter examples to confirm or reject claims about the properties of mathematical concepts; d) Can visualize sizes and quantities; e) Differentiates between mathematical concepts based on their properties, and recognizes similar concepts and the relations between them; f) Interprets different situations (including new ones) using mathematical concepts.

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5 **Mathematical concept:** a mental representation of a mathematical object (e.g. number, set, function, geometric solid and shape, plain, straight line, etc.), which reflects the main properties and relations.

6 **Representation:** presentation of a mathematical concept e.g. using concrete aids, graphic material, symbols, tables, computer simulations, etc.

7 **Size:** the result of measurement, which is expressed with a number and unit of measurement (e.g. $|AB|$ = 7.5 cm; p = 54 cm^2, etc.).

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.5. Knows and uses appropriate procedures[8] and tools[9] under different circumstances

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Uses successful procedures during play and when solving simple mathematical tasks; b) Learns about and researches various mathematical situations by: observing, one-to-one-corresponding, comparing, sorting, organizing, counting elements, etc.	a) Learns about and researches[10] various mathematical situations by: observing, one-to-one-corresponding, comparing, sorting and organizing elements. b) Solves mathematical tasks and problems by counting, measuring, collecting and presenting data; by drawing; by properly expressing sizes and quantities; by carrying out arithmetic procedures, taking into account the properties of arithmetic operations; c) Uses his/her own procedures when solving; d) Checks if the results of the implemented procedures are correct; e) Uses various aids and instruments.	a) Knows and uses different mathematical procedures when researching mathematical situations and solving tasks; b) Chooses appropriate procedures which lead to a solution; c) Uses his/her own procedures when solving; d) Checks if the results of the implemented procedures are correct; e) Chooses and uses appropriate tools for solving, expressing and communicating.	a) Knows and uses different mathematical procedures when researching unknown situations and solving tasks; b) Chooses appropriate procedures which lead to a solution; c) Uses his/her own procedures when solving; d) Checks if the results of the implemented procedures are correct; e) Chooses and uses appropriate tools for solving, expressing and communicating.	a) Knows and uses different mathematical procedures when researching unknown situations and solving tasks; b) Chooses appropriate procedures which lead to a solution; c) Uses new (his/her own) procedures when solving; d) Checks if the results of the implemented procedures are correct; e) When carrying out various activities, the learner effectively uses different tools or aids and takes their limitations into account.

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8   **Procedure:** a form of systematic and prudent work, action or thinking to achieve a goal (e.g. an arithmetic procedure or algorithm; cognitive procedures: observing, comparing, organizing, sorting, etc.; mathematical procedures: counting, measuring, presenting data, solving equations, etc.).

9   **Tool:** a geometric tool, measuring aids and instruments, computer programs, etc.

10   **Research:** in this context it is meant as creative work or activity with which we wish to broaden and improve our knowledge. We use it to establish or confirm facts; to determine and test the results of past work; to solve new or existing problems; to develop new theories, etc.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.6. Predicts and assesses results, substantiates claims, procedures and decisions

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Assesses which data are necessary; b) Predicts what will happen based on his/her own experience; c) Checks whether the solutions are correct, recognizes wrong solutions, and corrects them.	a) Assesses which data are necessary and sufficient in a mathematical situation or task; b) Predicts solutions based on his/her own experience; c) Assesses the suitability of implemented procedures when solving tasks; d) Checks whether the solutions are correct, recognizes wrong solutions, and corrects them.	a) Assesses which data are necessary and sufficient in a mathematical situation or task; b) Predicts solutions based on his/her mathematical knowledge and experience; c) Assesses the suitability of the chosen and implemented procedures when solving tasks; d) Evaluates the solutions obtained, suggesting corrections and improvements; e) Finds an example for his/her claim.	a) Assesses which data are necessary and sufficient in a mathematical situation or task; b) Predicts solutions based on his/her mathematical knowledge and experience and the data obtained; c) Assesses the suitability of the choice and implementations procedures when solving tasks; d) Evaluates the solutions obtained, assesses their suitability, and suggests corrections and improvements; e) Formulates his/her own mathematical claims, tests and substantiates them.	a) Assesses which data are necessary and sufficient in a mathematical situation or task; b) Predicts solutions based on his/her mathematical knowledge and experience and the data obtained; c) Assesses the suitability of the choice and implementations procedures when solving tasks; d) Evaluates the solutions obtained and assesses whether they are correct, sensible or suitable; corrects inappropriate solutions and suggests improvements; e) Formulates mathematical claims and hypotheses, and tests them (proves or refutes them); f) Substantiates mathematical claims at an appropriate level of strictness.

1st BUILDING BLOCK:

Mathematical thinking, understanding and use of mathematical concepts, procedures and strategies; communication as the basis of mathematical literacy

1.7. Uses different strategies when solving mathematical problems[11]

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	**SECOND EDUCATIONAL CYCLE ages 9-12**	**THIRD EDUCATIONAL CYCLE ages 12-15**	
a) Uses known strategies to solve challenges (e.g. trial and error, finding the odd one out, classification – appropriate for the development stage); b) Uses procedural knowledge to solve challenges, finding different paths to the solutions and multiple solutions to the problem; c) Forms various questions based on the given challenges; d) *Experiences solving challenges as a creative activity.*	a) Uses known strategies (appropriate for the development stage) to solve (routine) mathematical problems; b) Uses procedural knowledge to solve diverse (routine) mathematical problems (closed, open, with too much data, insufficient data, inconsistent data, with multiple solutions, without solutions, with an absurd solution); c) Forms different questions and similar tasks based on the given mathematical situations; d) Assesses the suitability of the implemented strategies when solving problems; e) *Experiences solving mathematical problems as a challenge and a creative activity.*	a) Uses known strategies (appropriate for the development stage) to solve mathematical problems; b) Uses procedural knowledge to solve diverse mathematical problems (closed, open, with too much data, insufficient data, inconsistent data, with multiple solutions, without solutions, with an absurd solution); c) Forms different questions and similar problems based on the given mathematical situations or problems; d) Assesses the suitability of the chosen strategies when solving problems; e) *Experiences solving mathematical problems as a challenge and a creative activity.*	a) Uses different strategies (e.g. trial and error, systematic testing, special cases) to solve mathematical problems; b) Uses procedural knowledge when solving diverse mathematical problems (closed, open, with too much data, insufficient data, inconsistent data, with multiple solutions, without solutions, with an absurd solution), when investigating[12] and discovering[13]; c) Forms different questions and similar problems based on the given mathematical situations or problems; d) Assesses the suitability of the chosen strategies when solving problems; e) *Experiences solving mathematical problems as a challenge and a creative activity.*	a) Uses sensible strategies (e.g. trial and error, backward thinking, systematic testing, special cases, analogy) to solve mathematical problems; b) Uses procedural knowledge (e.g. inductive reasoning, generalization, deductive reasoning) when solving diverse mathematical problems (closed, open, with too much data, insufficient data, inconsistent data, with multiple solutions, without solutions, with an absurd solution), when investigating and discovering; c) Forms different questions and new problems based on the given mathematical situations or problems; d) Assesses the suitability of the chosen strategies when solving problems; e) *Experiences solving mathematical problems as a challenge and a creative activity.*

11 **Problem:** is an initiative or challenge (task, situation, question) that requires an original solution, but the path to the solution is not given/known to the learner and he/she must look for it using his/her own thought processes.

12 **Investigation:** a primary and secondary school discussion of problem situations with unclear goals (we investigate tasks or challenges that do not specify what exactly we have to determine and how we can reach the solutions).

13 **(Learning by) discovery:** it is a more or less independent approach to solving and researching a problem, where the teacher maintains the learners' interest in solving it, provides them with appropriate support, and guides them.

2nd building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.1. Discusses diverse real life problems[14] (problems that do not require mathematical modelling)

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
a) Detects and defines a mathematical problem in a life situation; b) Illustrates the situation using concrete material and describes it using everyday language; c) Participates in designing a problem-solving plan; d) Designs and uses an appropriate mathematical strategy to solve the problem; e) Describes (partial and final) solutions in context.	a) Detects and defines a mathematical problem in a life situation; b) Illustrates the situation using concrete material and describes it using mathematical language; c) Guided by the teacher, the learner designs his/her own problem-solving plan and presents it; d) Designs and uses an appropriate mathematical strategy to solve the problem and solves the problem; e) Presents and thinks about how sensible the (partial and final) solutions are in context.	a) Recognizes a mathematical problem in a life situation and expresses it using mathematical language; b) Designs his/her own problem-solving plan and presents it; c) Designs and uses a sensible mathematical strategy to solve the problem and solves the problem; d) Presents, interprets and evaluates the (partial and final) solutions in context.	a) Recognizes a mathematical problem in a life situation and expresses it using mathematical language; b) Designs his/her own problem-solving plan and presents it; c) Designs and uses sensible mathematical strategies to solve the problem and solves the problem; d) Presents, interprets and evaluates the (partial and final) solutions in context.	a) Recognizes a mathematical problem in a life situation and expresses it using mathematical language; b) Designs his/her own problem-solving plan and presents it; c) Designs and uses sensible mathematical strategies to solve the problem and solves the problem; d) Presents, interprets and evaluates the (partial and final) solutions in context.

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14 **Life problem:** is a challenge (task, question, situation) that requires an original solution and a different way of solving it using one's thought processes. The context of a life problem stems from life or the everyday (e.g. a part of a newspaper article, the results of a study or a scientific paper, a news item, advertisement, etc.) and the data have not been didactically adapted to the learners' prior knowledge or development stage).

2ⁿᵈ building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.2. Discusses situations using mathematical modelling[15]

2.2.1. Transfers a situation to the mathematical context

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
	a) Participates in describing a (personal) life problem in the mathematical language; b) Participates in presenting the situation using mathematical means and in forming the problem question.	a) Participates in describing a (personal, social) life problem in the mathematical language; b) Presents the situation using mathematical means and forms the problem question.	a) Recognizes that he/she will be able to mathematically model the given situation; b) Describes a *(personal, social, technical)* problem in the mathematical language; c) Recognizes the quantities, mathematical concepts and relationships in the situation being discussed, and decides on their relevance; d) Simplifies the situation to enable a mathematical treatment; e) Presents the situation using mathematical means and forms problem questions in a mathematical context.	a) Recognizes that he/she will be able to mathematically model the given situation; b) Describes a *(personal, social, technical, scientific)* problem in the mathematical language; c) Recognizes the quantities, mathematical concepts and relationships in the situation being discussed, and decides on their relevance; d) Simplifies the situation to enable a mathematical treatment; e) Presents the situation mathematically *(with concepts represented in different ways; with procedures, figures, etc.)* and forms problem questions in a mathematical context.

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15 **Mathematical modelling:** is a form of solving a life problem through research, which involves an in-depth understanding of the context and the derivation of hypotheses which are important for finding a solution and which lead to generalized conceptual solutions or a model. The problem contains a great deal of data, which are often incomplete and must be organized; we must also decide which of them we will apply.

2nd building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.2. Discusses situations using mathematical modelling[15]

2.2.2. Designs mathematical models[16] for the given situation

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
		a) Participates in designing a model, defining the variables, and formulating the hypotheses; b) Participates in making the model by using appropriate mathematical and technological tools.	a) When designing the model, the learner defines the variables, formulates the hypotheses, and states the model's limitations; b) Chooses a suitable type of model *(empirical, simulation, theoretical, algorithmic, etc.)* according to the given situation; c) Recognizes and writes down the relationships between the selected variables or suggests a mathematical structure for the given situation *(e.g. a function rule, graph, linear equation, system of linear equations, diagram, table, geometric object, image, description or the like)*; d) Uses appropriate mathematical and technological tools to make the model.	a) When designing the model, the learner defines the variables, formulates the hypotheses, and states the model's limitations; b) Decides on the type of model *(empirical, simulation, theoretical, algorithmic, etc.)* and chooses a suitable one; c) Recognizes and writes down the relationships between the selected variables or suggests a mathematical structure for the given situation *(e.g. a function rule, graph, linear equation, system of linear equations, diagram, table, geometric object, conic section, image, description or the like)*; d) Uses appropriate mathematical and technological tools to make the model.

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16 **Mathematical model:** is a special way of mathematically presenting the discussed non-mathematical object or phenomenon using mathematical language (e.g. we use direct proportion as a model when shopping; a geometric sphere as a model when discussing a ball). A mathematical model is not the illustration of mathematical concepts with other concepts (e.g. illustrating a line segment with a thin stick).

2nd building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.2. Discusses situations using mathematical modelling[15]

2.2.3. Uses mathematical models

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
	a) Participates in describing the given model; b) Follows the problem-solving procedure according to the given model and carries out individual steps; c) Describes mathematical solutions in context.	a) Describes the given model and presents it; b) Uses the given models; c) Takes into account the characteristics of the context *(appropriate units, accuracy, rounding)*; d) Interprets the mathematical solutions *(calculations obtained with the model)* in context.	a) Describes the given models and his/her own models using different mathematical representations; b) Uses the given models and his/her own models; c) Explains the model[17] *(discerns the variables, the functional relations, the result from the given model)* and takes into account the characteristics of the context *(appropriate units, accuracy, rounding)*; d) Makes use of technological tools *(calculator, computer tables, various programs, web applications)* when using the model; e) Knows and uses model simulation techniques *(e.g. computer tables, programming, programs for working with functions, dynamic geometry programs)*; f) Interprets the mathematical solutions *(calculations obtained with the model)* in context.	a) Describes the given models and his/her own models using different mathematical representations; b) Uses the given models and his/her own models; c) Explains the model *(discerns the variables, the functional relations, the result from the given model)* and takes into account the characteristics of the context *(appropriate units, accuracy, rounding)*; d) Makes use of technological tools *(measuring aids, calculation and graphical representation aids, etc.)* when using the model; e) Knows and uses model simulation techniques *(e.g. computer tables, programming, programs for working with functions, dynamic geometry programs)*; f) Interprets the mathematical solutions *(calculations obtained with the model)* in context.

2nd building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.2. Discusses situations using mathematical modelling[15]

2.2.4. Evaluates mathematical models

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	SECOND EDUCATIONAL CYCLE ages 9-12	THIRD EDUCATIONAL CYCLE ages 12-15	
		a) Describes the suitability of the model under different circumstances; b) Tests the model's usability based on new data and circumstances.	a) Discusses the suitability *(reasonableness, correctness, accuracy)* of the model under different circumstances *(e.g. discussion of boundaries, discussion of hypotheses, neglected quantities)*; b) Tests the model's usability based on new data, examples and situations; c) Makes a more suitable model after determining the flaws of the given model; d) Compares different models *(e.g. according to accuracy, scope of application, ease of use)*.	a) Discusses the suitability *(reasonableness, correctness, accuracy)* of the model under different circumstances *(e.g. discussion of boundaries, discussion of hypotheses, neglected quantities)*; b) Tests the model's usability based on new data, examples and situations; c) Makes a more suitable model after determining the flaws of the given model; d) Compares different models *(e.g. according to accuracy, scope of application, ease of use)*.

2nd building block of mathematical literacy:

Problem solving in diverse contexts (personal, social, professional, scientific) that allow a mathematical treatment

2.3. Understands mathematical practices[17] in different contexts

PRE-SCHOOL EDUCATION ages 1-6	BASIC EDUCATION			UPPER SECONDARY SCHOOL ages 15-19
	FIRST EDUCATIONAL CYCLE ages 6-9	**SECOND EDUCATIONAL CYCLE ages 9-12**	**THIRD EDUCATIONAL CYCLE ages 12-15**	
		a) Recognizes non-formal mathematical practices and describes them using mathematical language.	a) Recognizes non-formal mathematical practices and describes them using mathematical language.	a) Recognizes non-formal mathematical practices and describes them using mathematical language; b) Interprets mathematical practices within the context of the mathematical model; c) Recognizes and understands the importance of "non-mathematical factors" in mathematical practices *(e.g. the importance of tools, tradition, a user's mathematical knowledge, the broader context of activities).*

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17 **Mathematical practices:** use of mathematics in professional situations/work processes, where we use different procedures than in school mathematics (e.g. joiner, tiler, salesperson, etc.).

Mathematical literacy
Definition and building blocks