

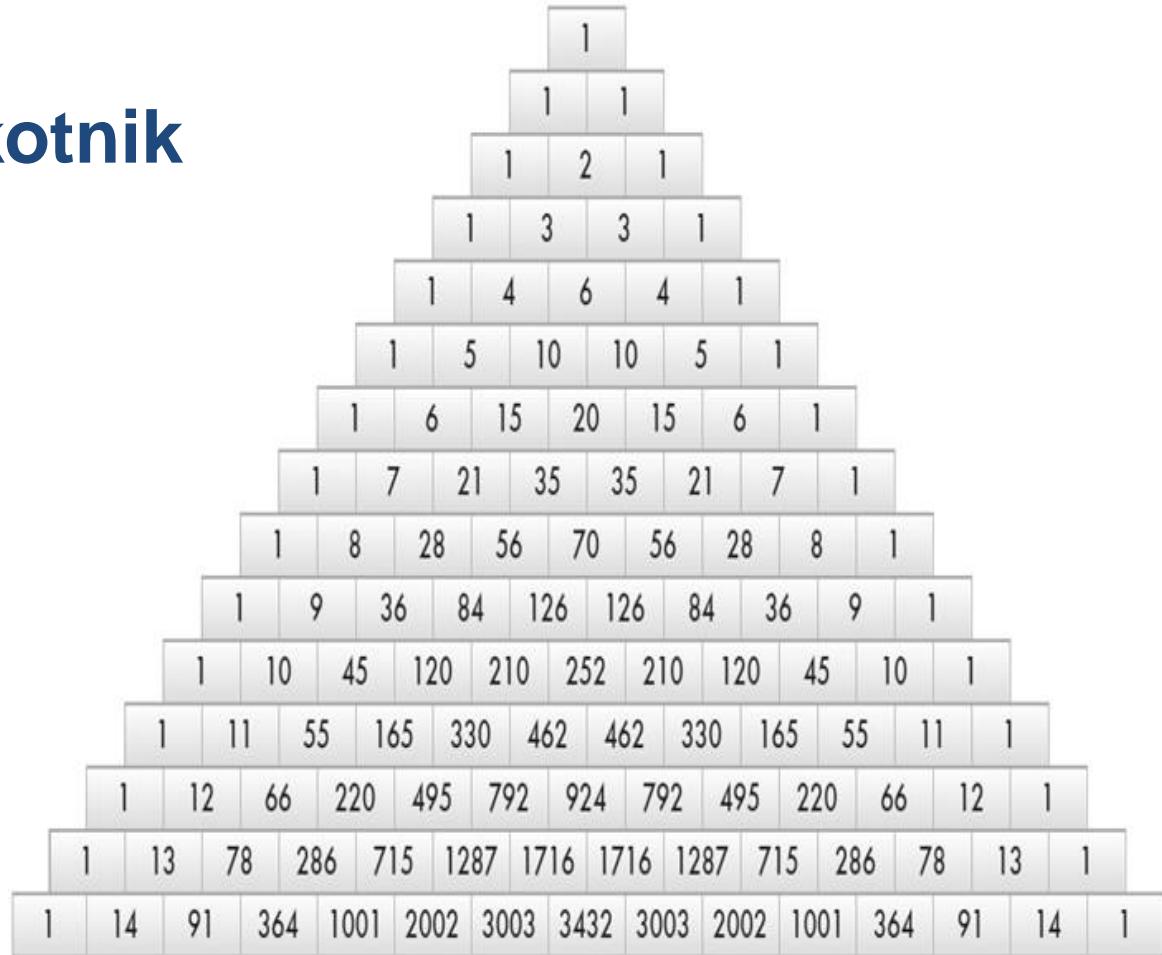
# **Od Pascalovega do Leibnizovega trikotnika**

**Mag. Sonja Rajh**

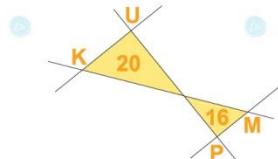
Zavod RS za šolstvo



# Pascalov trikotnik



<https://www.youtube.com/watch?v=YUqHdxxdbyM>



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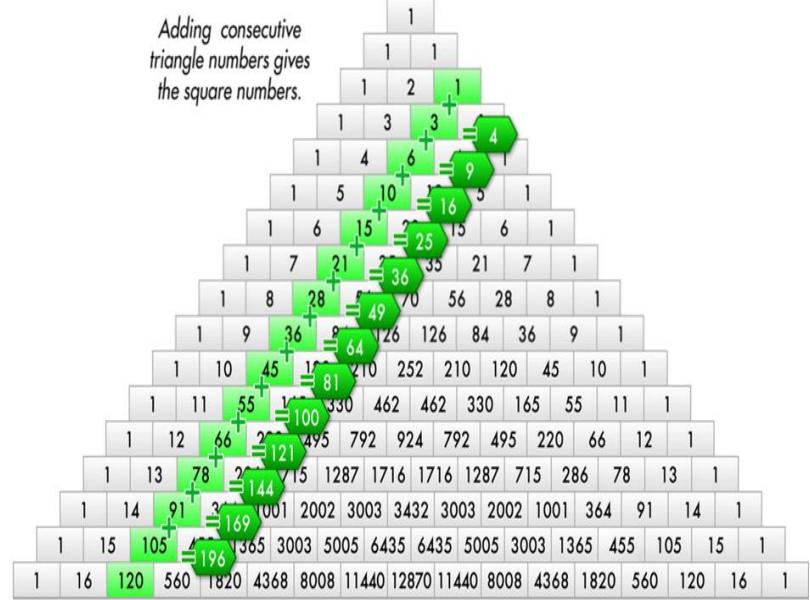
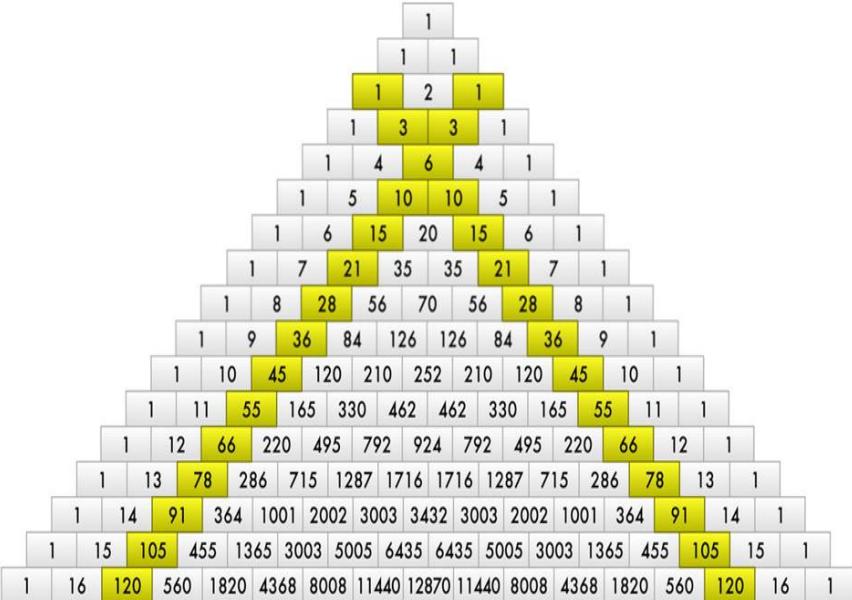
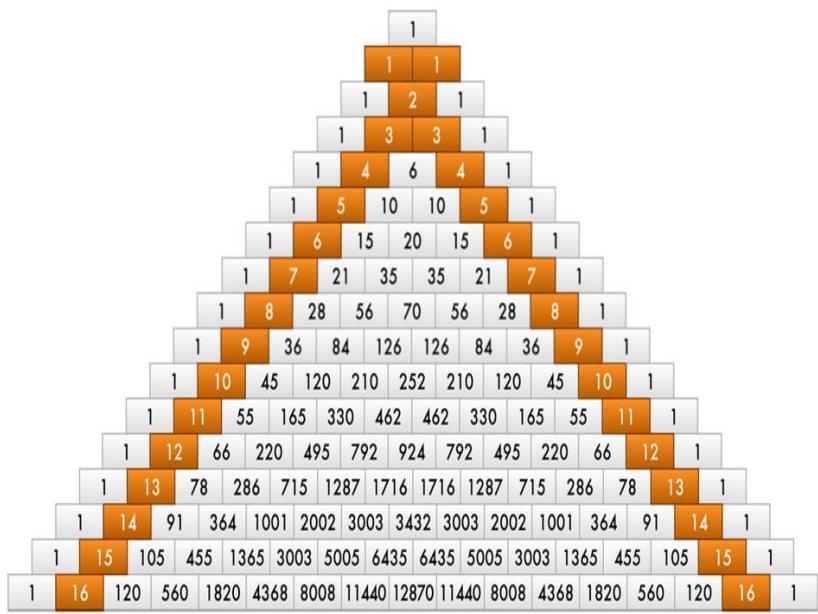
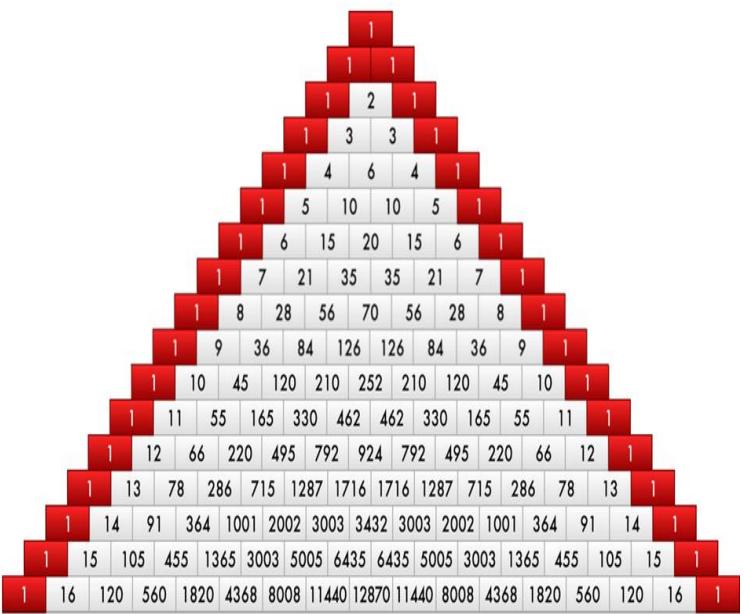
Zavod Republike Slovenije za šolstvo



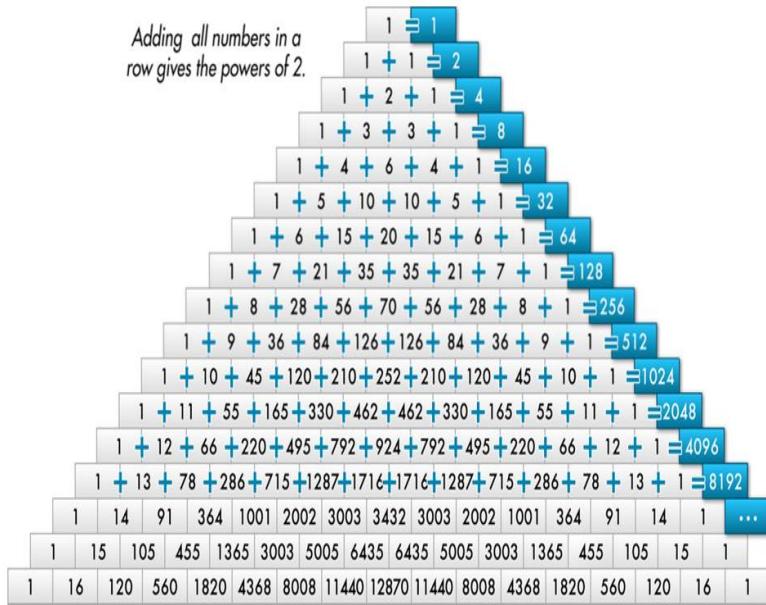
REPUBLIKA SLOVENIJA  
MINISTRSTVO ZA IZOBRAŽEVANJE,  
ZNANOST IN ŠPORT



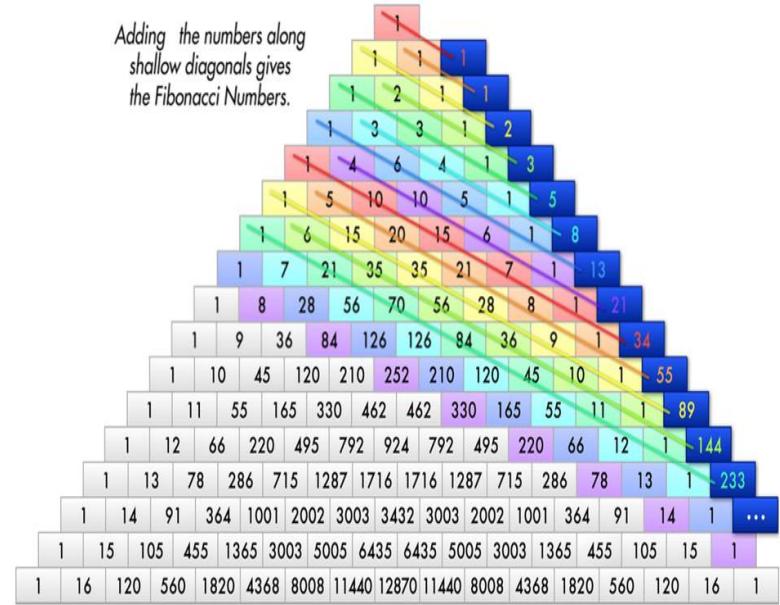
EVROPSKA UNIJA  
EVROPSKI  
SOCIALNI SKLAD  
NALOŽBA V VAŠO PRIHODNOST



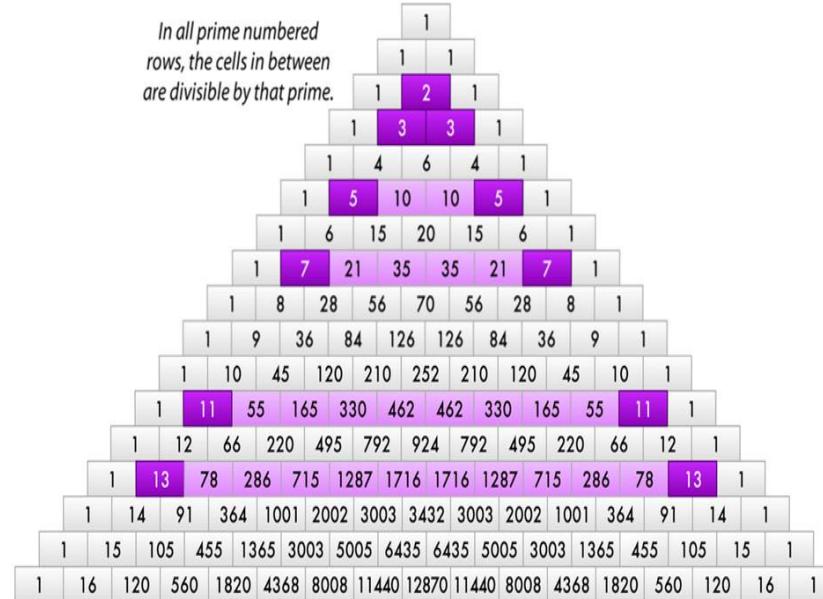
Adding all numbers in a row gives the powers of 2.



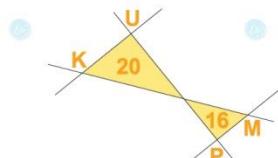
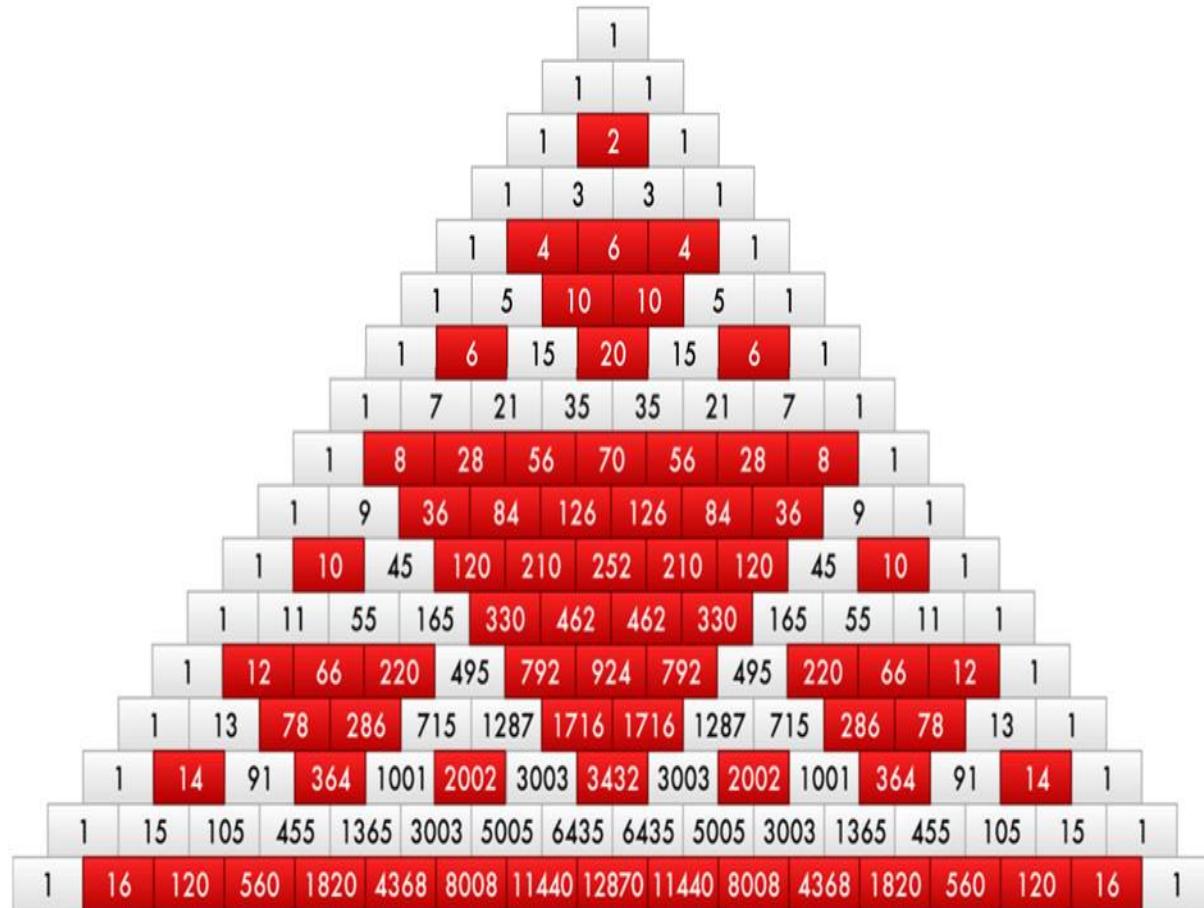
Adding the numbers along shallow diagonals gives the Fibonacci Numbers.



In all prime numbered rows, the cells in between are divisible by that prime.



V shemi Pascalovega trikotnika pobarvajte večkratnike števila 2.



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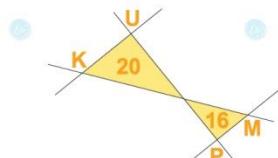
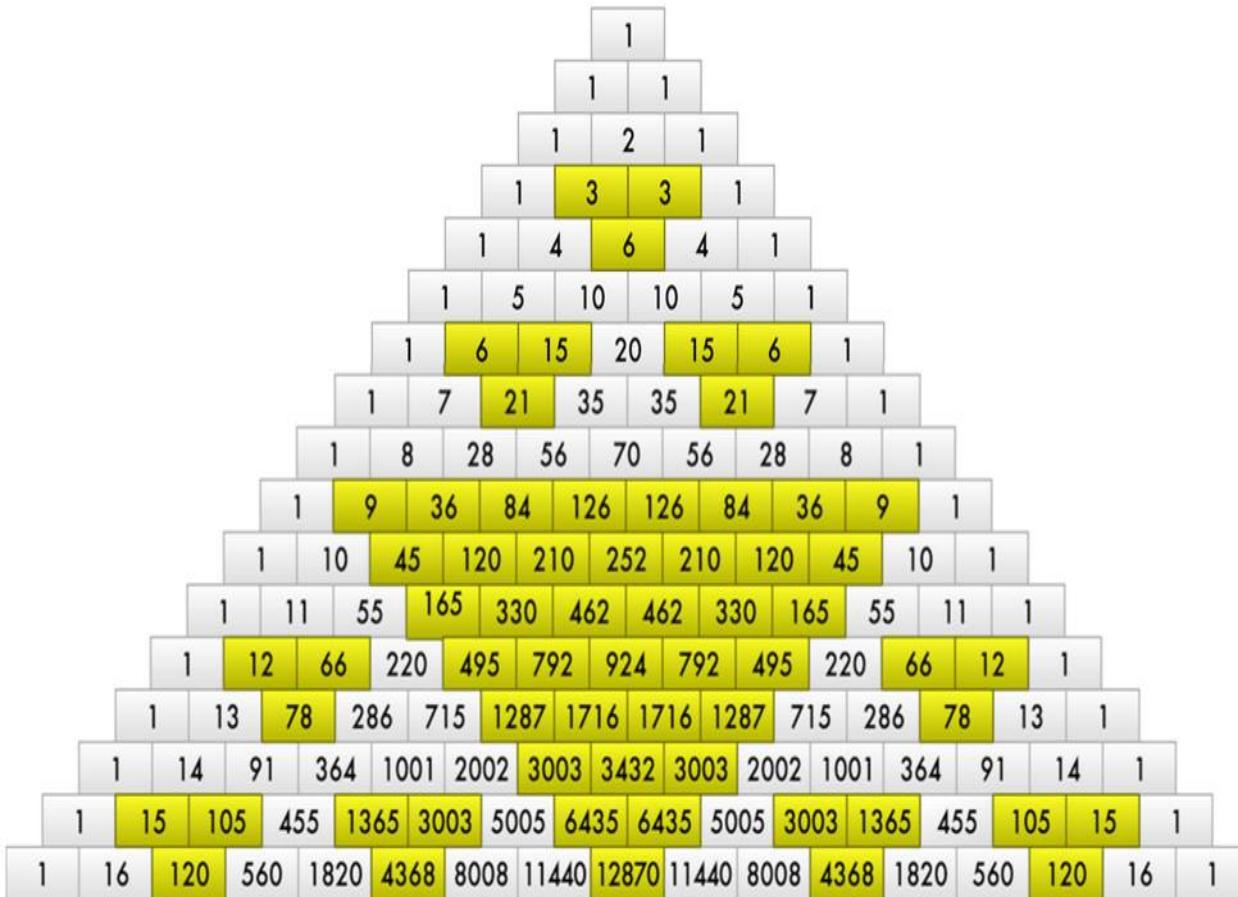


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V shemi Pascalovega trikotnika pobarvajte večkratnike števila 3.



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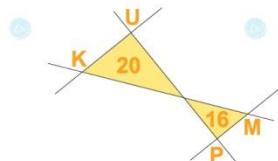
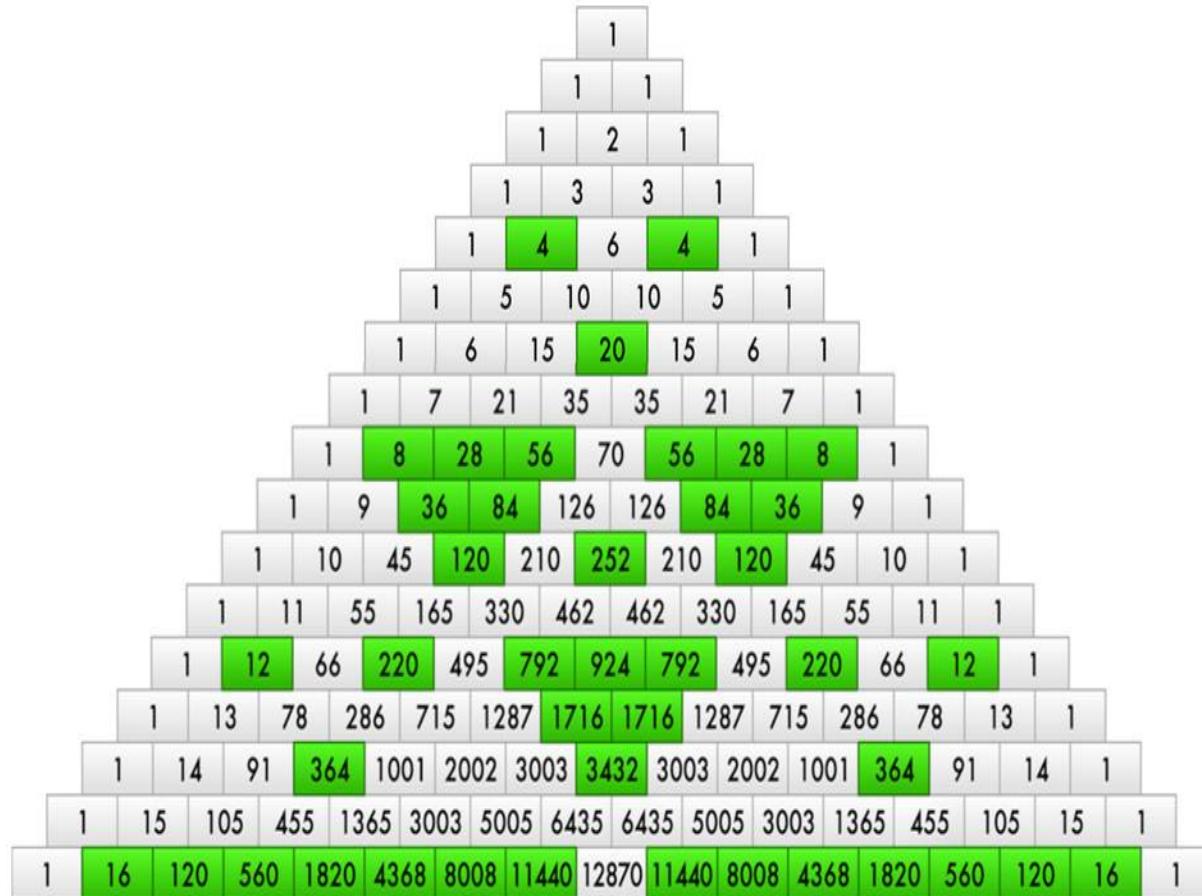


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V shemi Pascalovega trikotnika pobarvajte večkratnike števila 4.



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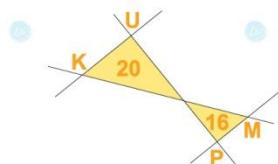
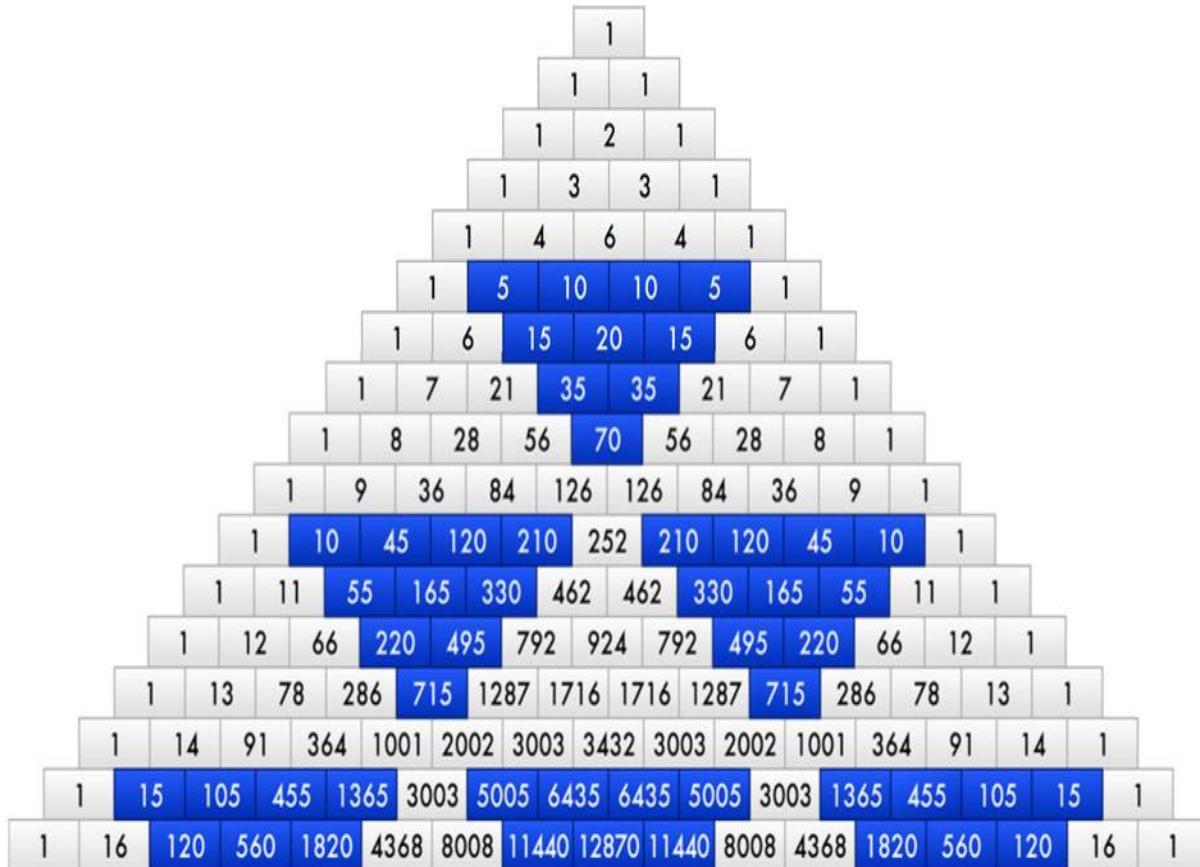


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V shemi Pascalovega trikotnika pobarvajte večkratnike števila 5.



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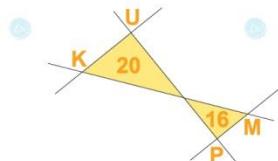
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SOCIALNI SKLAD  
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# Trikotnik Sierpinskega



[https://www.google.si/search?q=Trikotnik+Sierpinskega&biw=1178&bih=516&tbm=isch&tbo=u&source=univ&sa=X&ved=0ahUKEwjn\\_uqcm6bQAhUBCywKHUbhB14QsAQIIw&dpr=1.16#imgdii=MnRNIMdYQrP7CM%3A%3BMnRNIMdYQrP7CM%3A%3Bzft4qdur\\_EOTuM%3A&imgrc=MnRNIMdYQrP7CM%3A](https://www.google.si/search?q=Trikotnik+Sierpinskega&biw=1178&bih=516&tbm=isch&tbo=u&source=univ&sa=X&ved=0ahUKEwjn_uqcm6bQAhUBCywKHUbhB14QsAQIIw&dpr=1.16#imgdii=MnRNIMdYQrP7CM%3A%3BMnRNIMdYQrP7CM%3A%3Bzft4qdur_EOTuM%3A&imgrc=MnRNIMdYQrP7CM%3A)

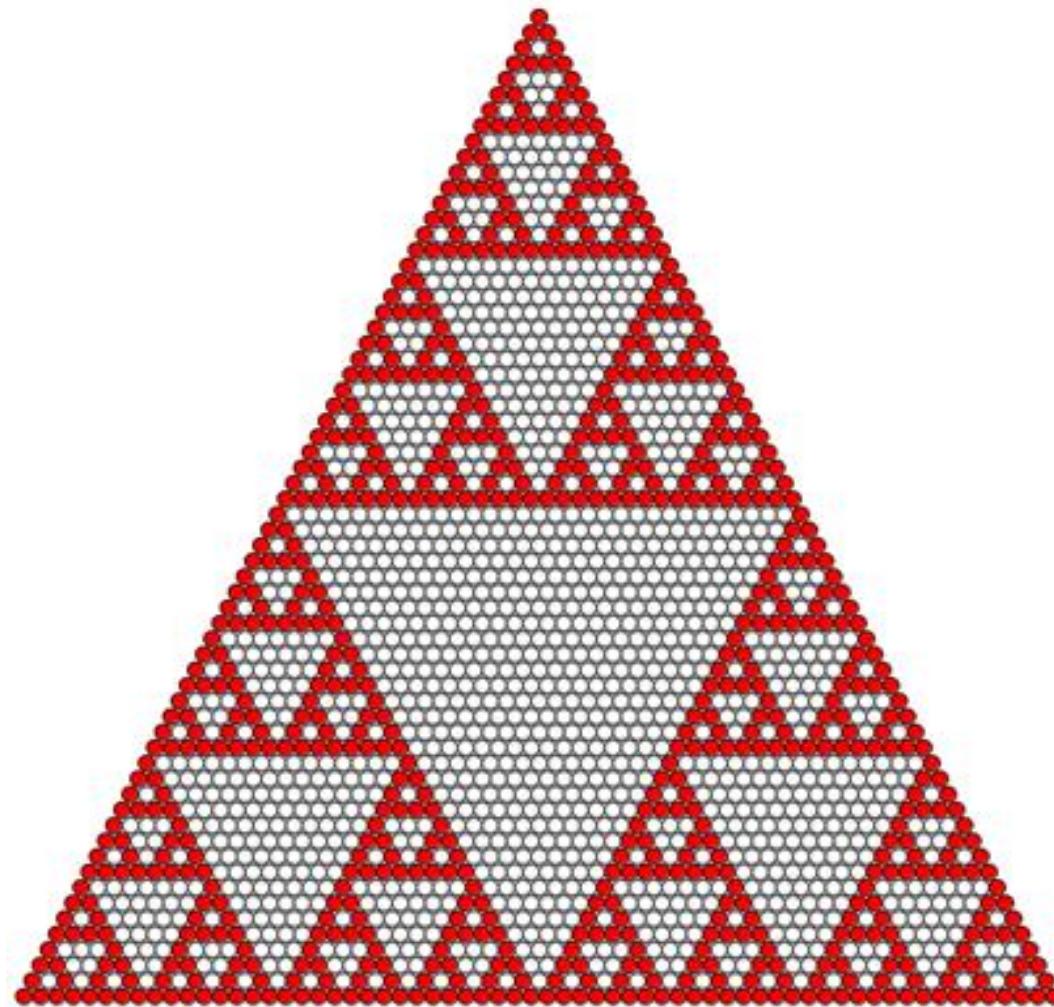
<http://math.bu.edu/DYSYS/>



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# Pascalov trikotnik

Ostanek pri  
deljenju z 2



Modulus

< 2 >

# Rows

< 64 >

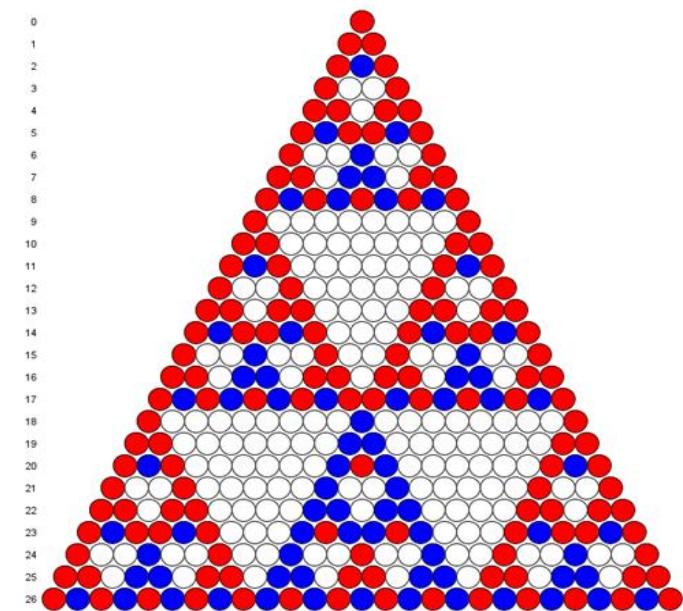
Show digits

Right triangle

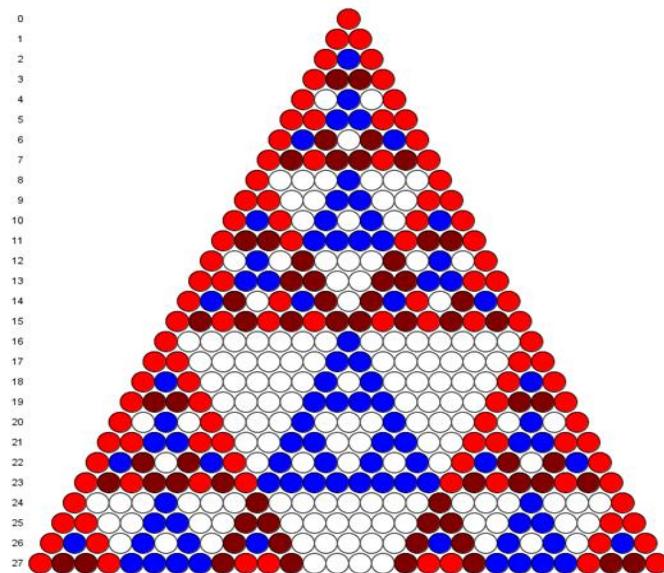
Pascal

Leibnitz

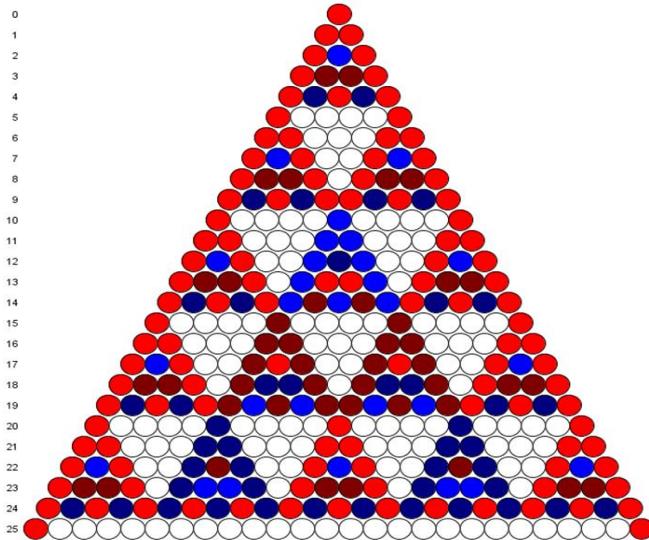
# Pascalov trikotnik



# Pascalov trikotnik



# Pascalov trikotnik



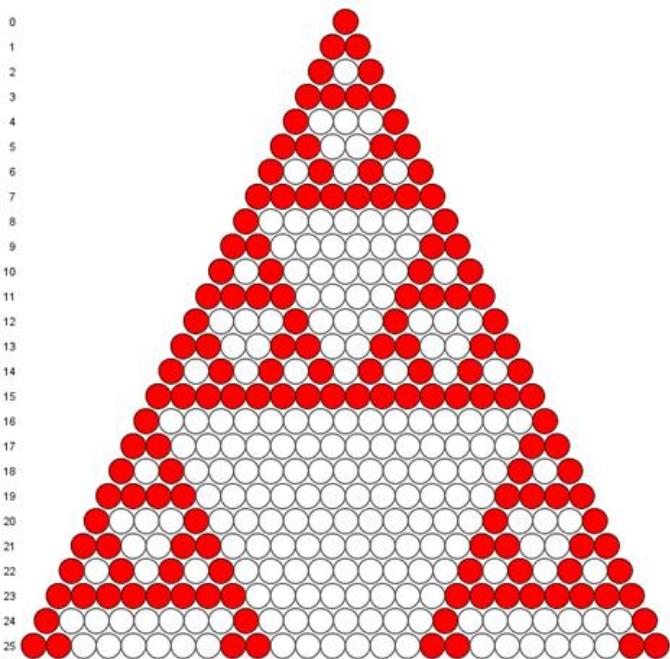
Modulus  # Rows

Show digits  Right triangle  Pascal  Leibnitz

1	1	1																						
1	2	1																						
1	3	3	1																					
1	4	1	4	1																				
1	0	0	0	0	1																			
1	1	0	0	0	1	1																		
1	2	1	0	0	1	2	1																	
1	3	3	1	0	1	3	3	1																
1	4	1	4	1	1	4	1	4	1															
1	0	0	0	0	2	0	0	0	0	1														
1	1	0	0	0	2	2	0	0	0	1	1													
1	2	1	0	0	2	4	2	0	0	1	2	1												
1	3	3	1	0	2	1	1	2	0	1	3	3	1											
1	4	1	4	1	2	3	2	3	2	1	4	1	4	1										
1	0	0	0	0	3	0	0	0	3	0	0	0	0	1										
1	1	0	0	0	3	3	0	0	3	3	0	0	0	1	1									
1	2	1	0	0	3	1	3	0	0	3	1	3	0	0	1	2	1							
1	3	3	1	0	3	4	4	3	0	3	4	4	3	0	1	3	3	1						
1	4	1	4	1	3	2	3	2	3	2	3	2	3	1	4	1	4	1						
1	0	0	0	0	4	0	0	0	1	0	0	0	4	0	0	0	0	1						
1	1	0	0	0	4	4	0	0	0	1	1	0	0	4	4	0	0	0	1					
1	2	1	0	0	4	3	4	0	0	1	2	1	0	0	4	3	4	0	0	1	2	1		
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1	4	1	4	1	4	1	4	1	4	1	4	1	4	1	4	1	4	1	4	1	4	1	4	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Ostanek pri  
deljenju s 5

# Pascalov trikotnik - praštevila



MODULUS

< 2 >

# Rows

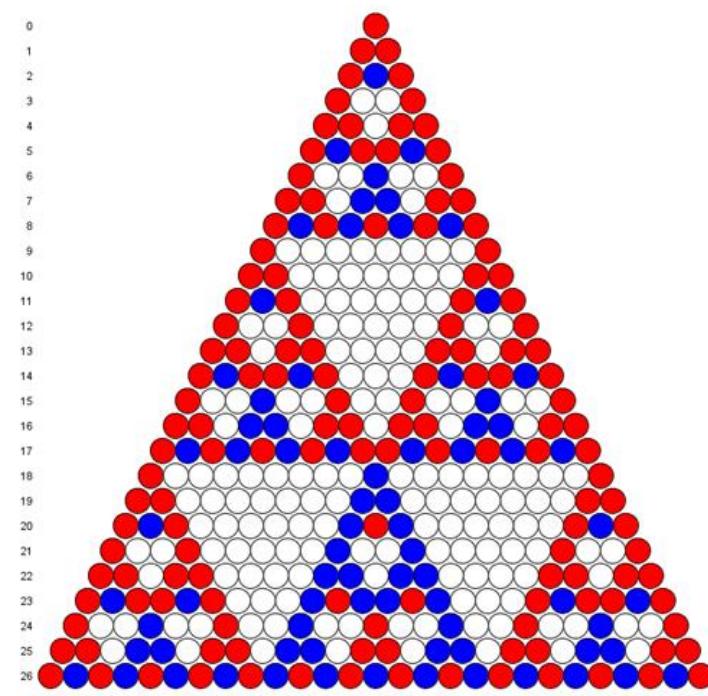
< 26 >

Show digits

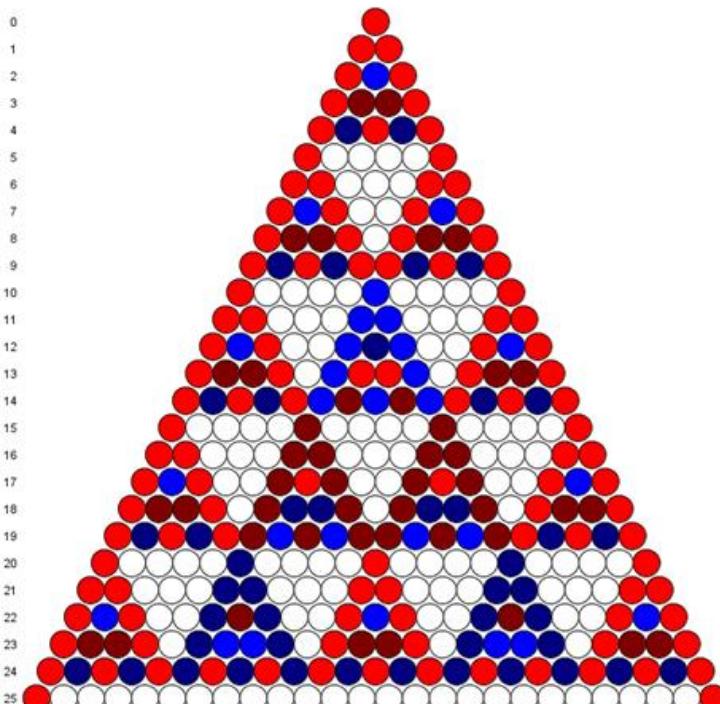
Right triangle

Pascal

Leibnitz



# Pascalov trikotnik - praštevila



Modulus

# Rows

Show digits

Right triangle

Pascal

Leibnitz

0

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

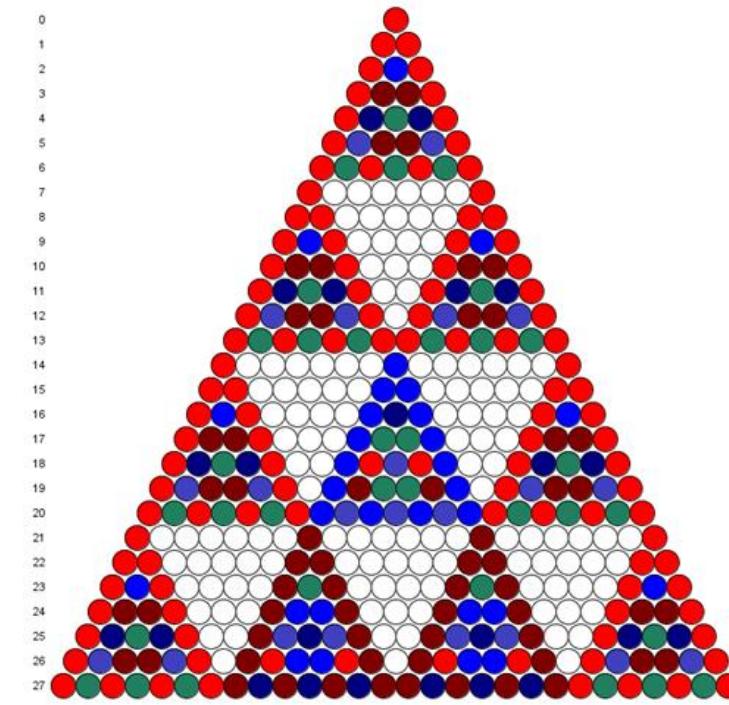
23

24

25

26

27



Modulus

# Rows

Show digits

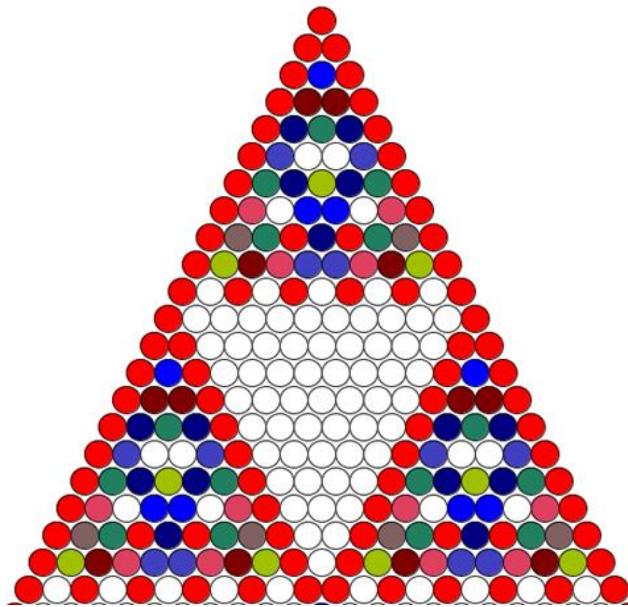
Right triangle

Pascal

Leibnitz

# Pascalov trikotnik - praštevila

0  
1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21

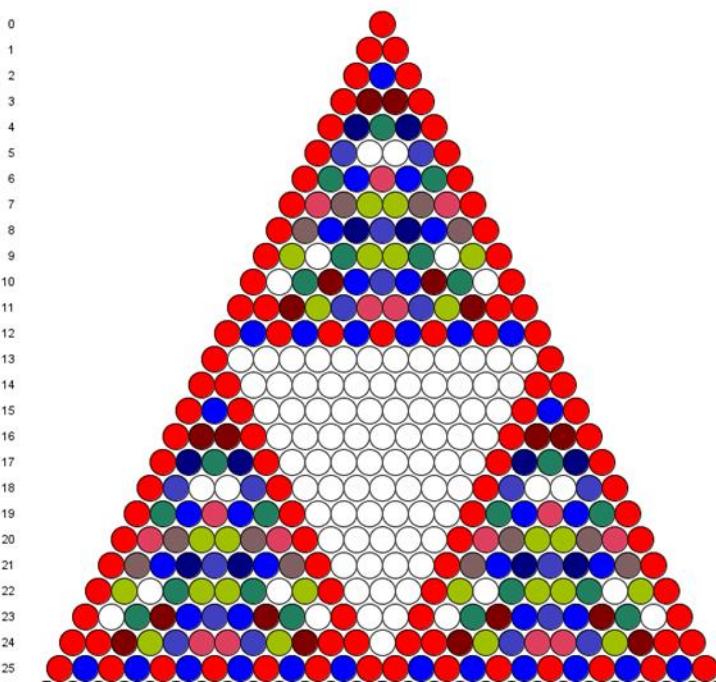


Modulus

# Rows

Show digits  
 Right triangle  
 Pascal

0  
1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21



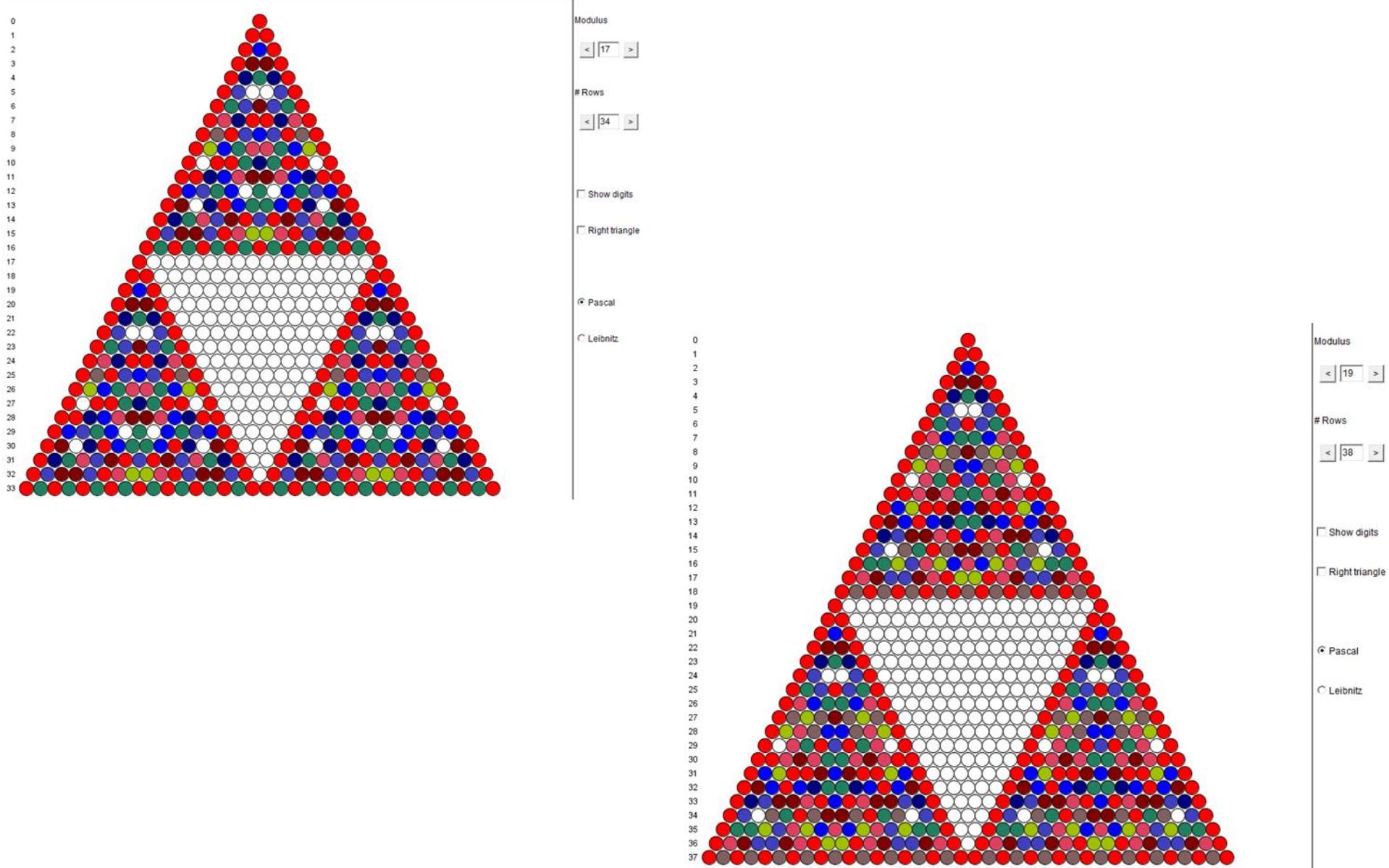
Modulus

# Rows

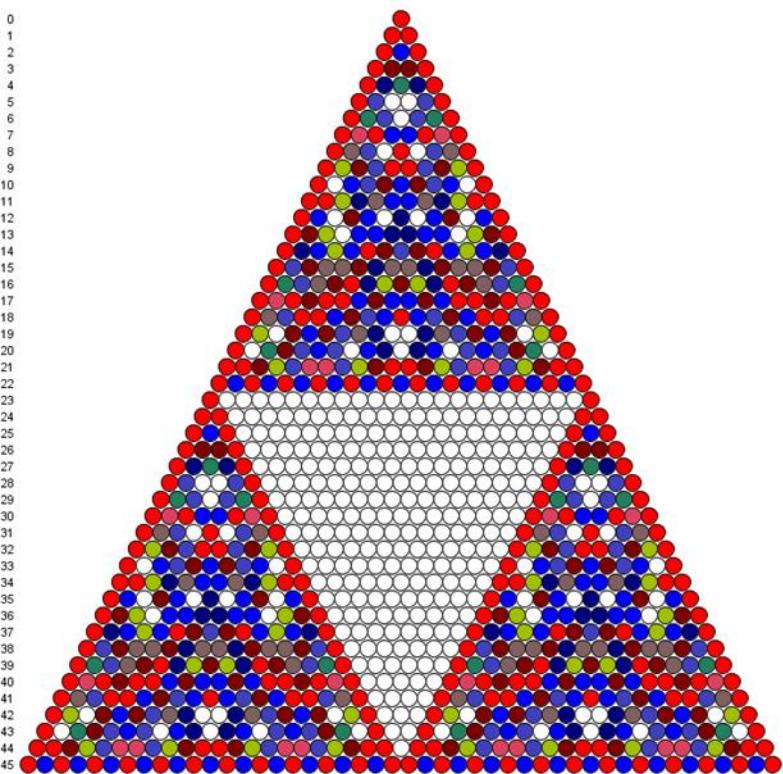
Show digits  
 Right triangle

Pascal  
 Leibnitz

# Pascalov trikotnik - praštevila



# Pascalov trikotnik - praštevila

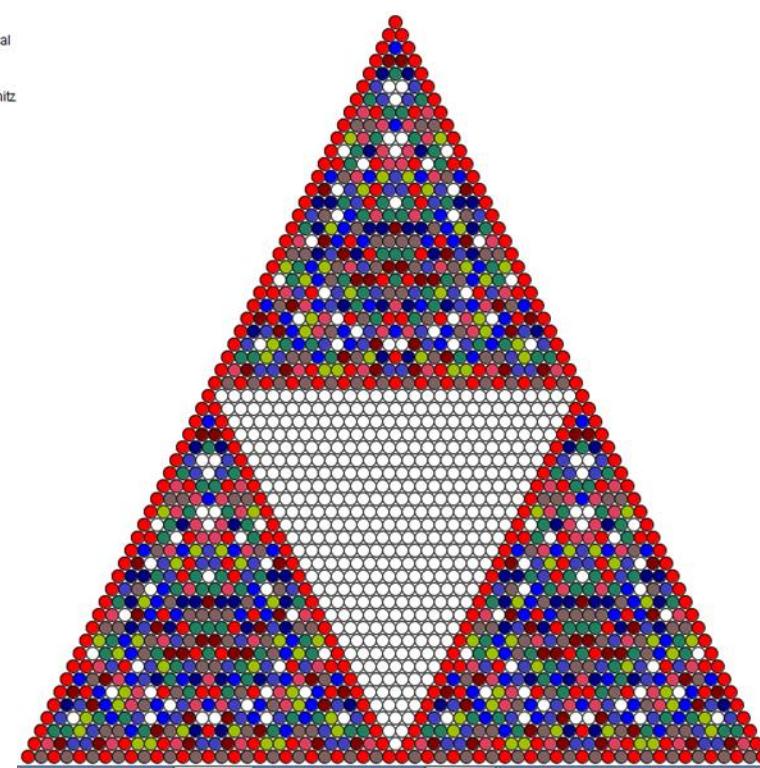


Modulus  
23

# Rows  
46

Show digits  
Right triangle

Pascal  
Leibnitz



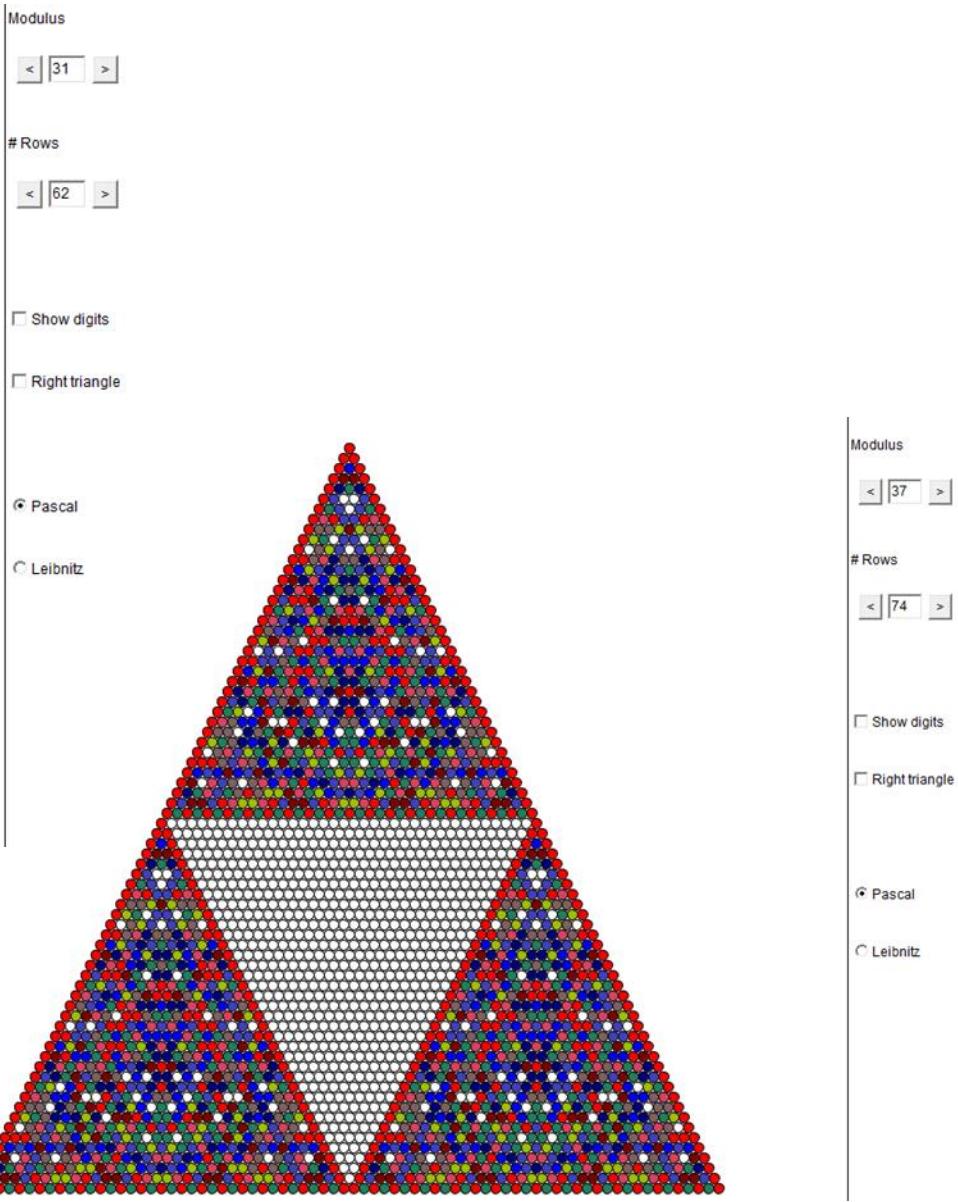
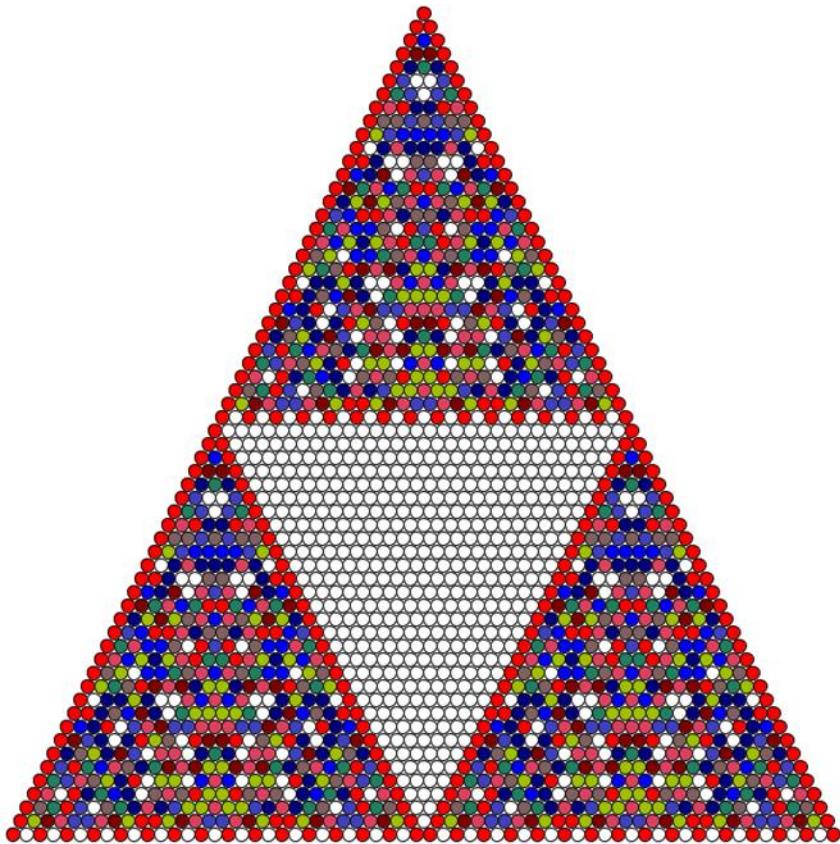
Modulus  
29

# Rows  
58

Show digits  
Right triangle

Pascal  
Leibnitz

# Pascalov trikotnik - praštevila

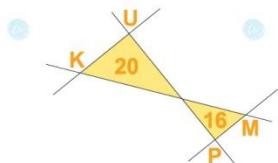


# Leibnizov harmonični trikotnik

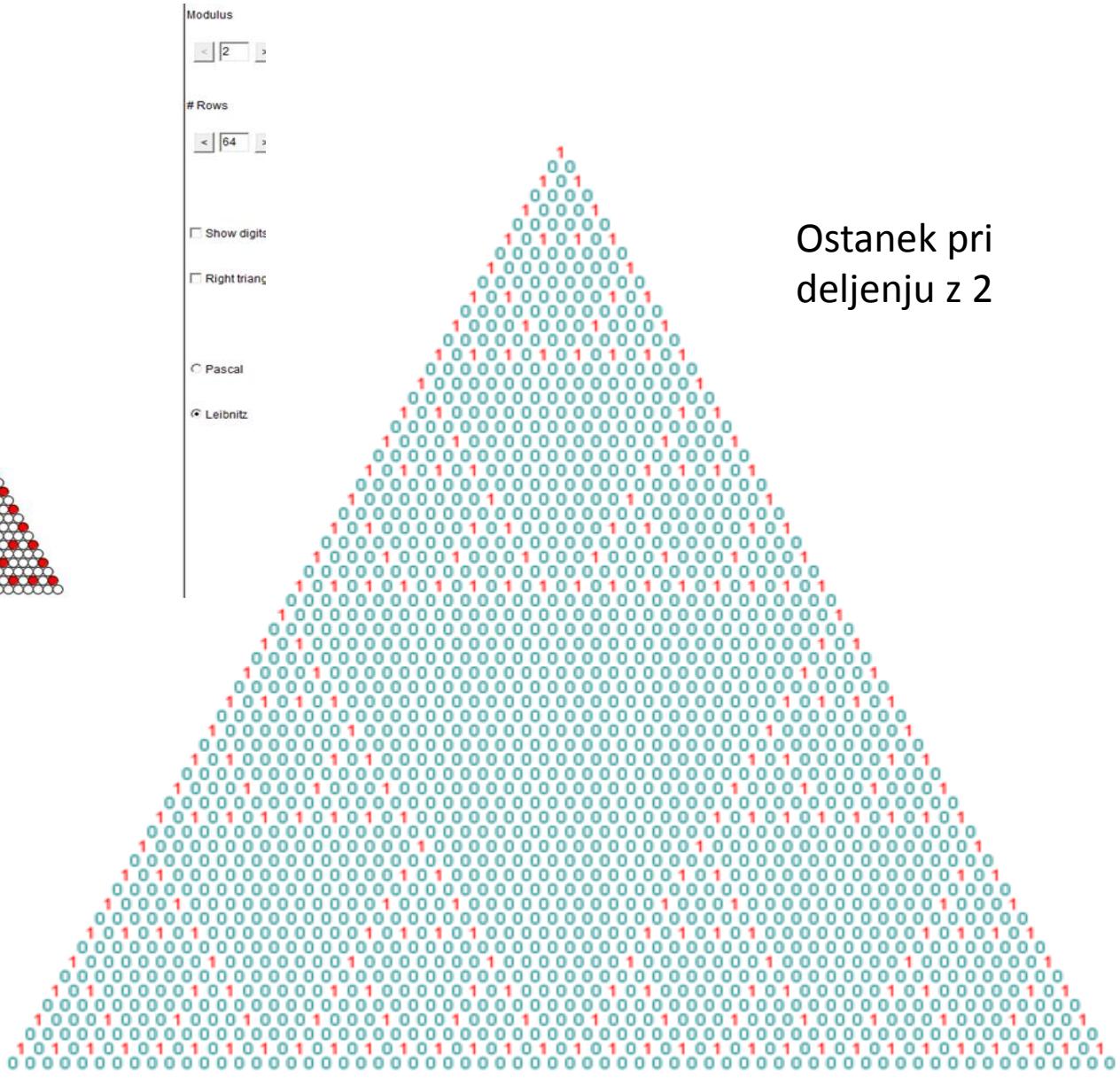
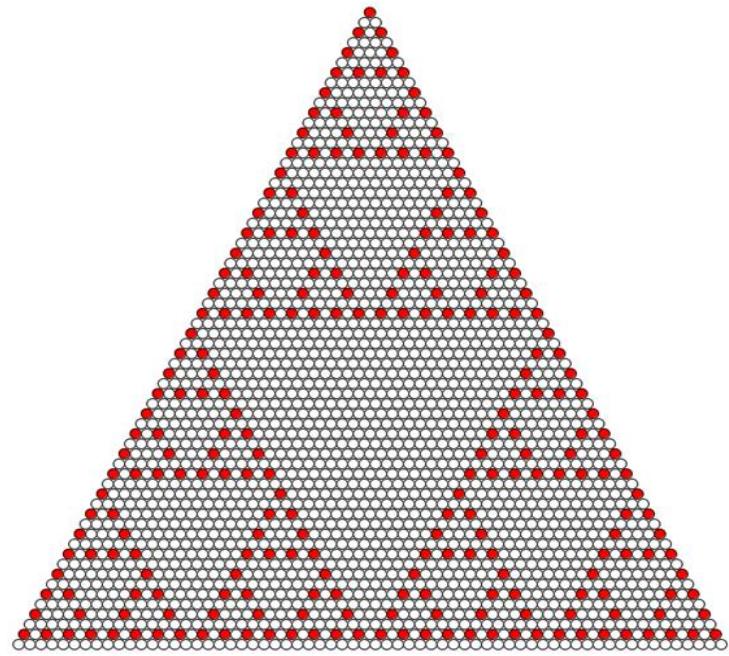
- $n$ -ta vrstica Leibnizovega harmoničnega trikotnika se prične in tudi konča z obratno vrednostjo števila  $n$ , torej z  $1/n$

- posamezno število v notranjosti Leibnizovega harmoničnega trikotnika je dobljeno kot vsota obeh števil pod njim

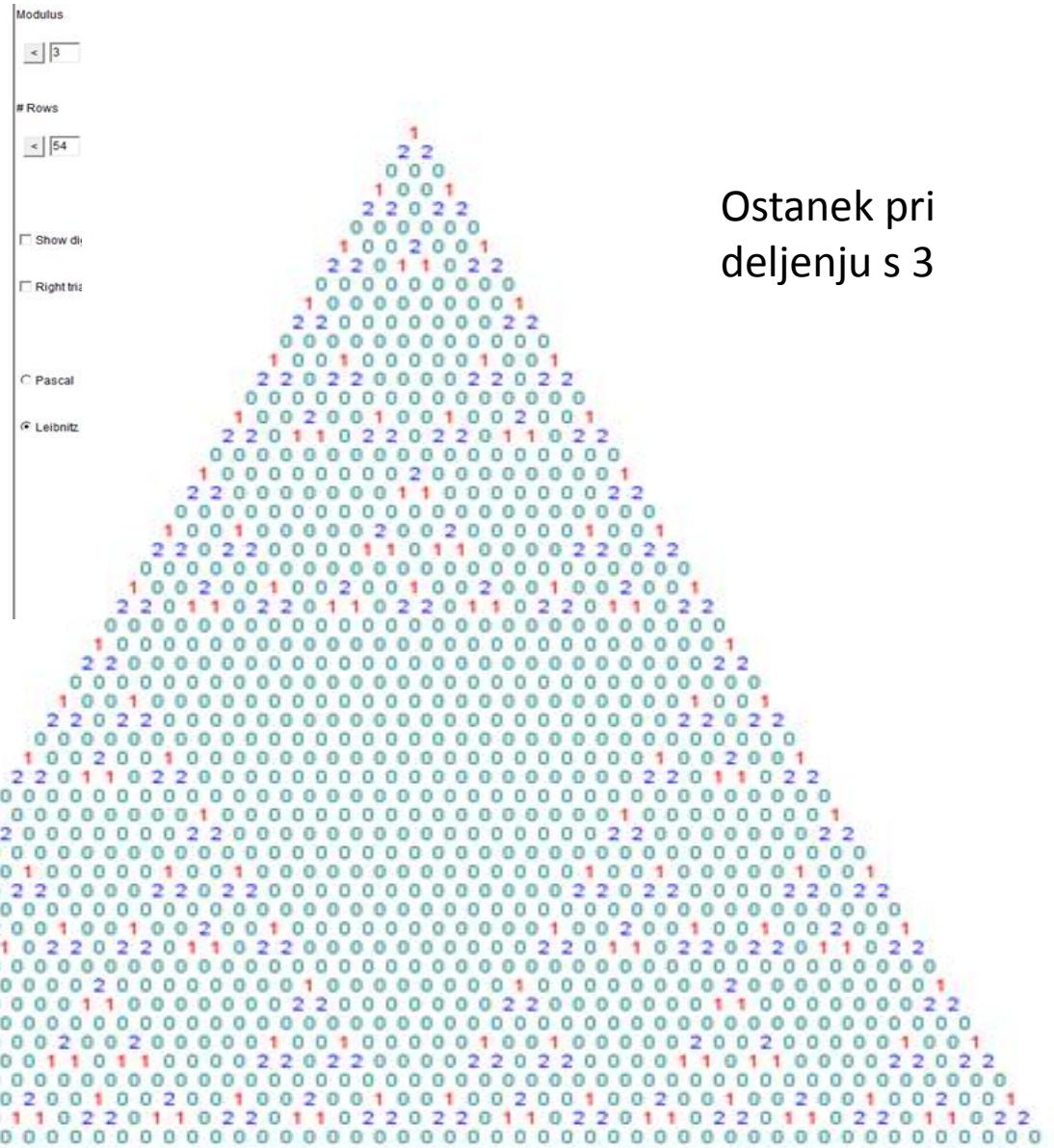
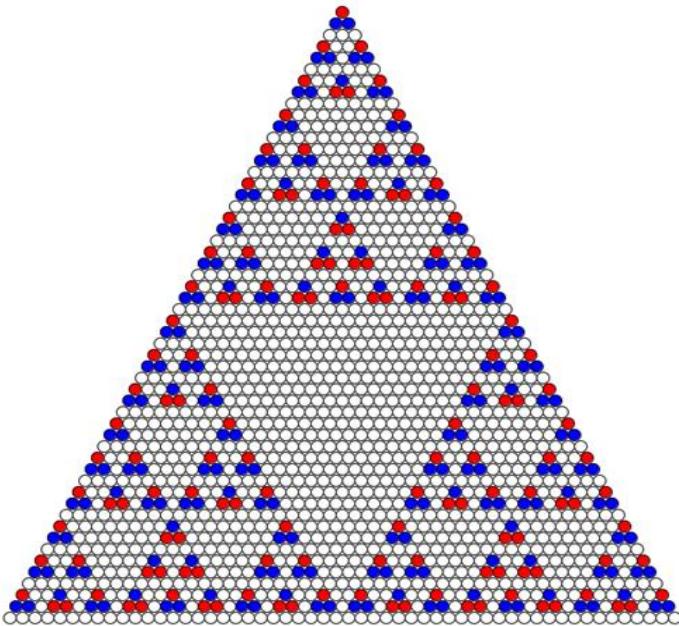
$$\begin{array}{ccccccccc} & & \frac{1}{1} & & & & & & \\ & & \frac{1}{2} & \frac{1}{2} & & & & & \\ & & & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & & & \\ & & & \frac{1}{4} & \frac{1}{12} & \frac{1}{12} & \frac{1}{4} & & \\ & & & \frac{1}{5} & \frac{1}{20} & \frac{1}{30} & \frac{1}{20} & \frac{1}{5} & \\ & & & \frac{1}{6} & \frac{1}{30} & \frac{1}{60} & \frac{1}{60} & \frac{1}{30} & \frac{1}{6} \\ & & & \frac{1}{7} & \frac{1}{42} & \frac{1}{105} & \frac{1}{140} & \frac{1}{105} & \frac{1}{42} & \frac{1}{7} \end{array}$$



# Trikotnik iz imenovalcev števil Leibnitzovega trikotnika

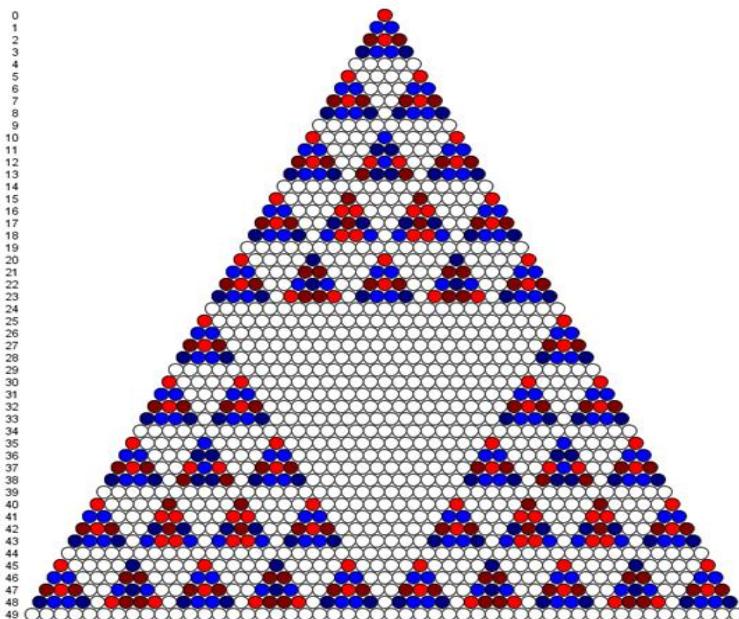


# Trikotnik iz imenovalcev števil Leibnitzovega trikotnika





# Trikotnik iz imenovalcev števil Leibnitzovega trikotnika



Modulus

# Rows

Show digits

Right triangle

Pascal

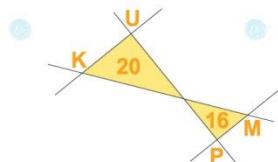
Leibnitz

Viri:

<https://www.youtube.com/watch?v=YUqHdxxdbyM>

<http://www.shodor.org/interactivate/activities/ColoringMultiples/>

<http://www.cut-the-knot.org/Curriculum/Combinatorics/LeibnitzTriangle.shtml>



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