An international initiative to stimulate research competences in math lessons

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Acquintance

- Lecturer, member of the educat
  of Antwerp, Belgium
- Member of the Editoral Board o
- Former teacher trainer in Hollar
  secundary and primary school t
- Experience as math teacher in
  black school

- Former competitor in and now t
  pupils) at the International Mat
- Member of the committee for "V
  initiative to stimulate research
- Introduced this initiative to Moj
  interesting for schools in Sloven

Photo on the front page of a Swedish newspaper

Introduction

- In general secondary education in Belgium, Holland, Germany, there is on the curriculum: “research competences” (since ca. 10 years)
- In Flanders on the curriculum:
  pupils can
  - deal with research problems by collecting, organize and processing information;
  - prepare, implement and evaluate a research project with a mathematical component;
  - report their results and confront them with other viewpoints.

Problems

- Very ambitious (can pupils formulate decent research questions in math on their own??)
- Teachers were (are) struggling
- (Math) inspectors are unsatisfied
- For every subject the research curriculum is identical, but mathematics is different/very specific:
  - survey in human science
  - experiment in chemistry/physics/biology
  - proof/counterexample in mathemics

Not possible with a series of fixed steps

- But nevertheless, by working on these competences:
  - pupils get also (close) to problem solving competences
  - pupils learn to find decent reasoning
  - pupils learn to put reasoning in a text/words
  - pupils learn working together

→ So, it is definitely worth pursuing these competences !!!

Wiskunde B

- In this context, Wiskunde B was born
- Originally at Freudenthal Institute (University of Utrecht, The Netherlands)
- To help teachers in secondary schools to work on the research competences
- Has become a contest for 17-18y pupils (working in groups of 3/4):
  - a whole day of math at school, prepared by an experienced team of professors, teachers and teacher trainers
  - one major subject/context
  - two main parts: exploratory part (to get some notions, vocabulary) and research part
  - at the end of the day: every group gives in a report with a text on both parts

Wiskunde B (in a nutshell)

- Participating schools
  - in Belgium, Holland, Germany, Slovakia
  - next year also Japan, Bulgaria,… and more to come?
- Schools can participate in the contest
  - a fixed day in november
  - teachers are prepared during one afternoon about the upcoming assignment (and needed material)
  - teachers do the first reading/selection of the reports
  - then evaluation at national (and international until 2013) level
  - closing event with best 10 teams of the competition
- Also a lot of schools do not participate, but use the assignment to work on the research competences
- More info/assignments of the past years, on the website: http://www.fi.uu.nl/wisbdag/ (questions not published before may)

Examples

- Topics:
  - Reflecting on mirrors
  - (Cr)easy
  - The final move
  - In the hands c
  - By the bend (t

Example: (Cr)easy Prep(rost)o-gibanje!

Subject: simple folding (left-right) and unfolding of a strip

Folding recipy: rlrl (use only lower case

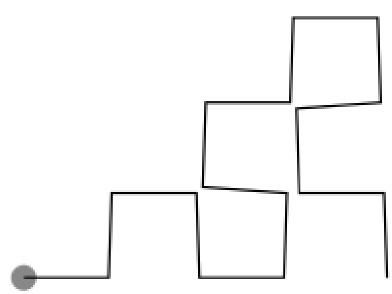
Angle (in degrees between 0 and 179):

88

walking pattern (WP) – symbolic description

Applet: provided by the organisation!

LRRLLLRRLRRRLLR



walking pattern (WP) - graph

Example of questions: (Cr)easy

- Exploratory questions:

1. - There are eight possibilities for a three-fold recipe. What are they?
   - Make an overview for the three-fold recipes (like the one above for two-fold recipes) both the folding recipes and their walking patterns.
   - You won't find the zigzag pattern below as a possible three-fold pattern. You could have known in advance. How?

2. Suppose you have a strip with 15 bends (from a 4-fold recipe)
   - Someone has made a walking pattern with an *R* in the positions 2, 7, 8 and 12. That is enough information to retrieve the whole walking pattern!
   - Determine the whole walking pattern (give the series of *R*s and *L*s).
   - What was the folding recipe?

Exploratory question 2

fold		walking pattern														
nr	value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	r								R							
2	l				L								R			
3	r		R				L				R				L	
4	l	L		R		L		R		L		R		L		R
		L	R	R	L	L	L	R	R	L	R	R	R	L	L	R

number of bends in the walking pattern: 1+2+4+8=15

Example of questions: (Cr)easy

- Research questions:

1. For every walking pattern, the first and the last straight piece are perpendicular to each other. Try to find a solid reasoning that shows this is always true.
2. Select a fixed number n (= 1, 2, 3, 4, 5, ...). For all walking patterns that you can make using the chosen n folds, the distance from start to finish is the same. Investigate why this is so. *Note*: that distance of course depends on the length of the strip you start with. It makes sense to start with a strip of length 2^n, so that all straight pieces will have length 1.
3. - Where are the possible end points of a walking pattern after n folds?
   - On the way from the starting point to the end point there may be points that are further away from the starting point than the end point is. Investigate what distances are possible. Can you express the maximum distance from the starting point to any point in the walking pattern in n?
4. - Can you prove that RRRR (or LLLL) is not possible in a walking pattern?
   - When do you run into points of contact that only occur later in the pattern?

Research questions 1 & 4

- With n-fold, you have exactly 2^n-1 bends → odd number of bends → first and last straight piece are perpendicular
- Last folding generates the odd places in the walking pattern, and these are alternating (see EQ2). Since exact two odd numbers occur in every consecutive set of four, it is impossible to have LLLL or RRRR.

fold		walking pattern														
nr	value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	r								R							
2	l				L								R			
3	r		R				L				R				L	
4	l	L		R		L		R		L		R		L		R
		L	R	R	L	L	L	R	R	L	R	R	R	L	L	R

- Starting from n=4, you always have RRR or LLL. Therefore, for n>4 also, and you get always more of them (think of anti-derivative)

Success

- Very positive feedback from teachers and pupils
- Teachers from other subjects notice the enthusiasm from the pu
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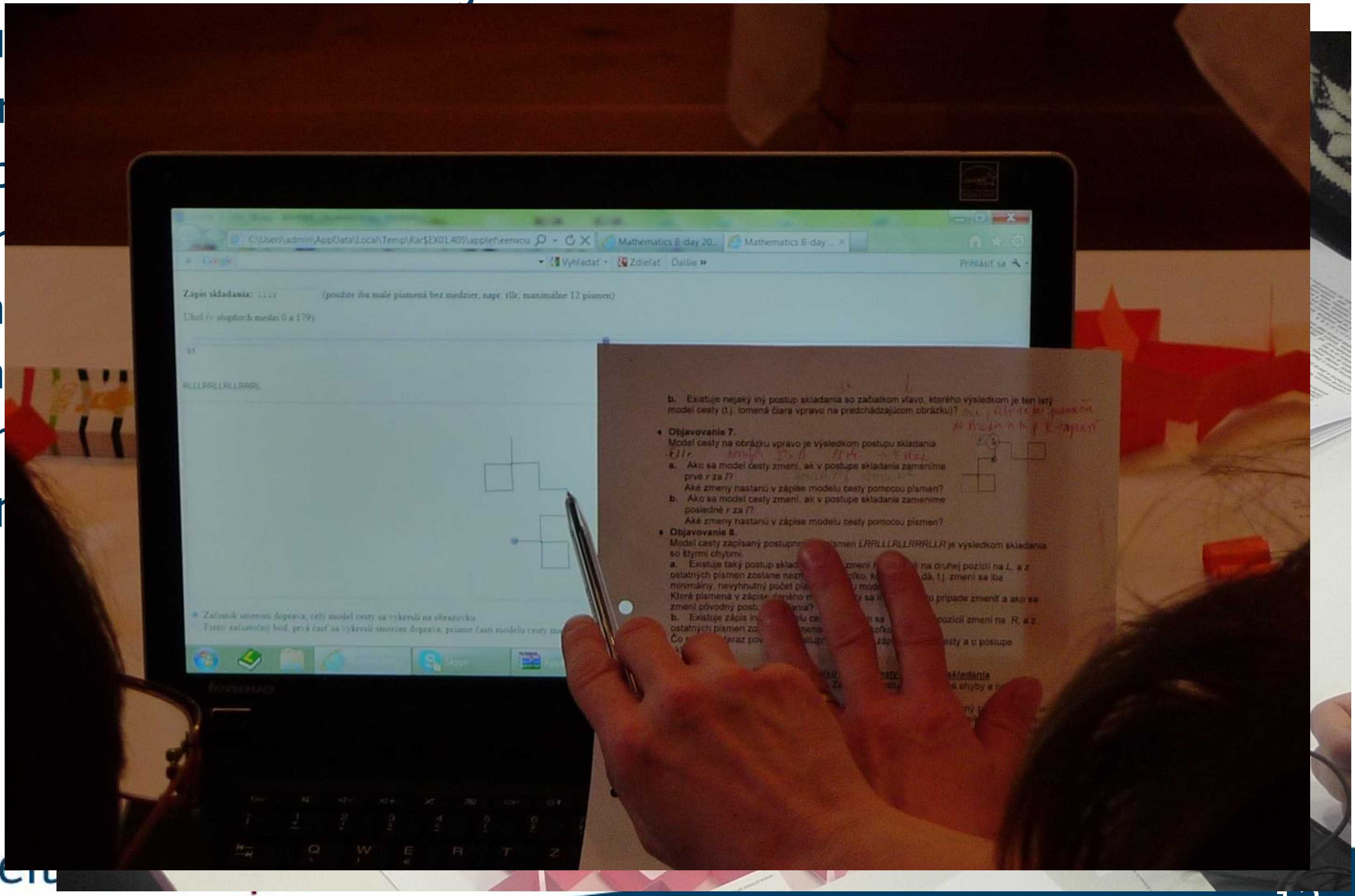


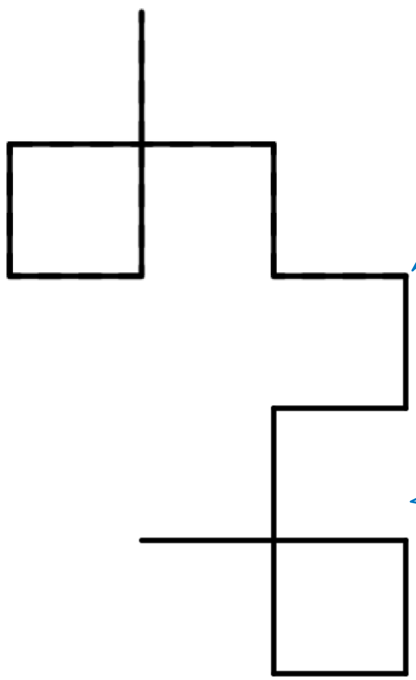
Questions?

- Does such an initiative is interesting for (your) Slovenian pupils?
- Does anyone share my enthusiasm on such math assignments?
- Do you think this kind of initiative is possible in a Slovenian context?
- Other questions ...(from the audience)?

Antisymmetry (EQ 1c)

folding recipy: lllr

walking pattern: $\boxed{\text{R L L L R R L}}$ L R L L R R R L

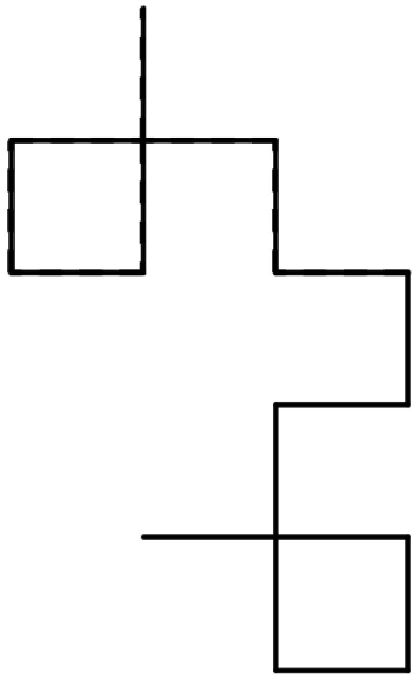


first fold: l (gives L)

this is a walking pattern as well (omit the first fold, gives llr)

Research question 1 (RQ1)

- first segment $\perp$ last segment?
- Use folding basics (see EQ1)
- different proof: odd number of bends of 90° (see EQ2)



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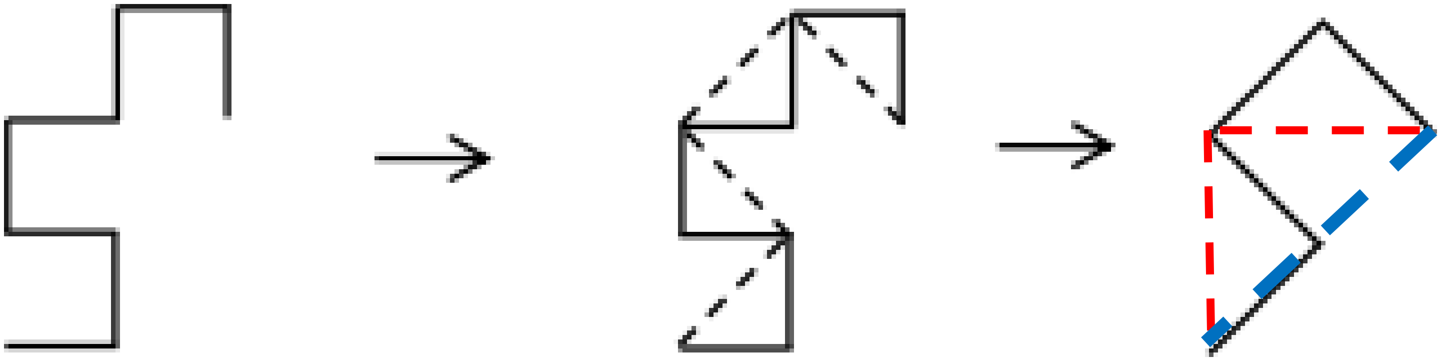
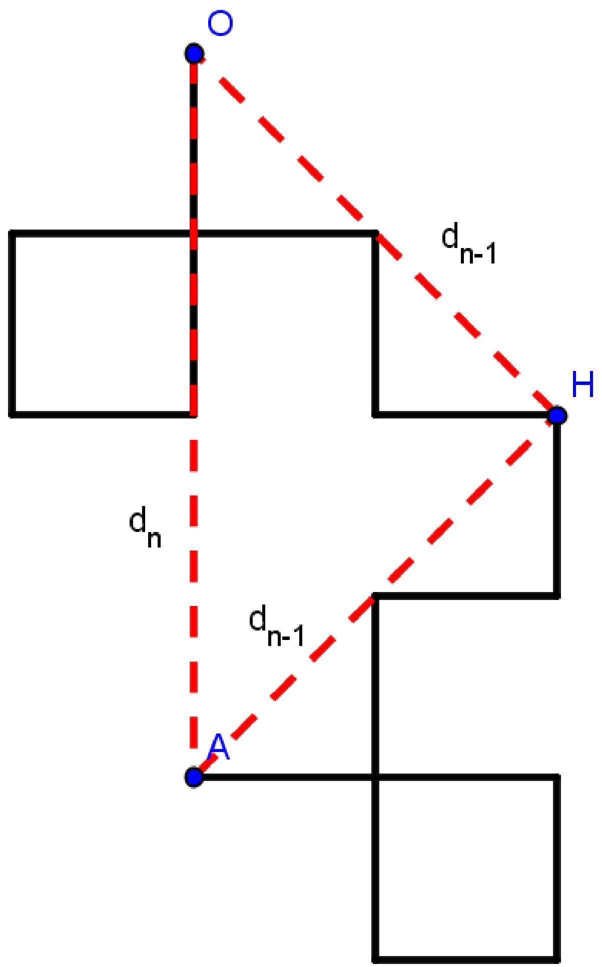
Recursive pattern: derivative



- corresponds to omitting the last fold
- WP of n folds $\rightarrow$ WP of n-1 folds
- some groups called it the derivative
- can be done more than once, until you end up with one segment
- → RQ 2: length between start and end point is always $\sqrt{2}^n = 2^{n/2}$.

Alternative for RQ2



If you keep the length of the segments constant, then

distance between begin and end point after n folds

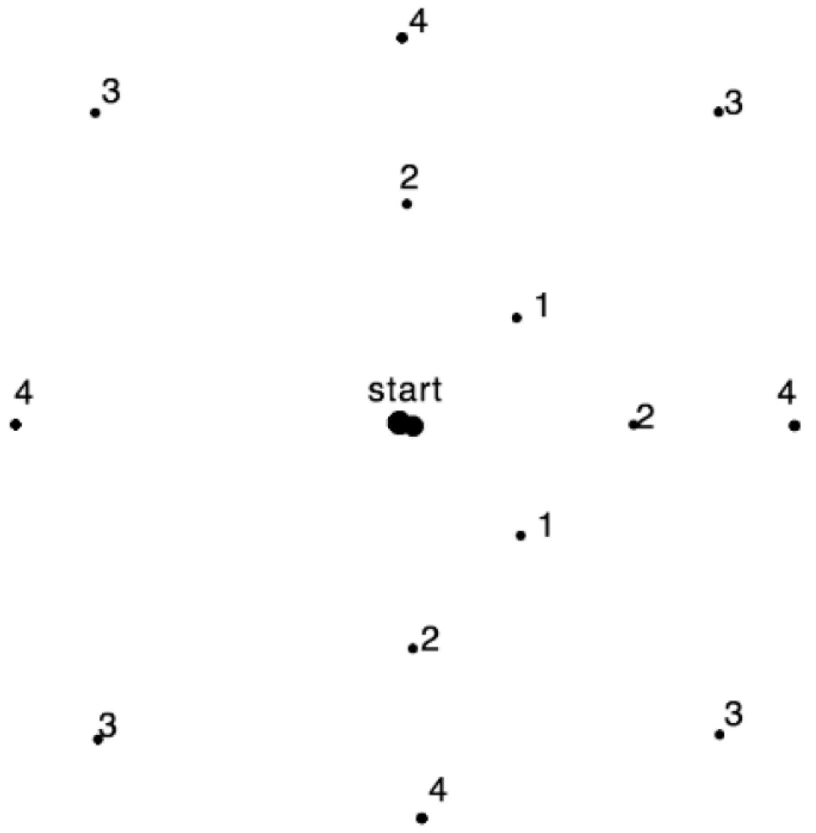
$=$

$\sqrt{2}$

times

distance between begin and end point after $n-1$ folds

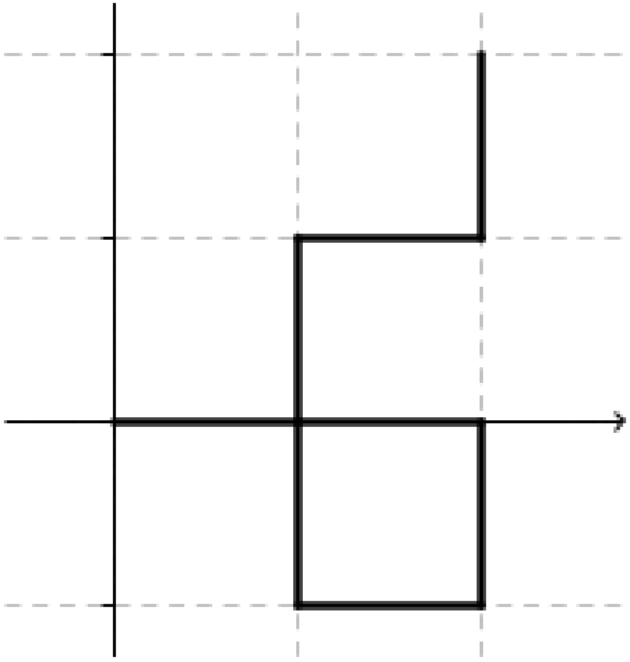
Endpoints (RQ3)



Folding with complex numbers

folding recipy: llr

walking pattern: RLLLRRL



point		vector		bend	
0	0				
1	1	E	1	R	-i
2	1-i	S	-i	L	i
3	2-i	E	1	L	i
4	2	N	i	L	i
5	1	W	-1	R	-i
6	1+i	N	i	R	-i
7	2+i	E	1	L	i
8	2+2 i	N	i		

Complex numbers

- By reasoning, you can find quite straight forward (by using the antisymmetry) that

$$p_n = (1 - i)^{\#r} \cdot (1 + i)^{\#l}$$